on the problems in "4 Trig functions/4.7 Miscellaneous"
Test results for the 69 problems in "4.7.1 (c trig)^m (d trig)^n.txt"
Problem 1: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a) \sin (2 b x+2 a)^{7} \mathrm{~d} x
$$

Optimal (type 3, 53 leaves, 4 steps):

$$
\frac{128 \sin (b x+a)^{9}}{9 b}-\frac{384 \sin (b x+a)^{11}}{11 b}+\frac{384 \sin (b x+a)^{13}}{13 b}-\frac{128 \sin (b x+a)^{15}}{15 b}
$$

Result(type 3, 110 leaves):
$\frac{35 \sin (b x+a)}{128 b}-\frac{35 \sin (3 b x+3 a)}{384 b}-\frac{21 \sin (5 b x+5 a)}{640 b}+\frac{3 \sin (7 b x+7 a)}{128 b}+\frac{7 \sin (9 b x+9 a)}{1152 b}-\frac{7 \sin (11 b x+11 a)}{1408 b}-\frac{\sin (13 b x+13 a)}{1664 b}$

$$
+\frac{\sin (15 b x+15 a)}{1920 b}
$$

Problem 2: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a) \sin (2 b x+2 a)^{5} d x
$$

Optimal(type 3, 40 leaves, 4 steps):

$$
\frac{32 \sin (b x+a)^{7}}{7 b}-\frac{64 \sin (b x+a)^{9}}{9 b}+\frac{32 \sin (b x+a)^{11}}{11 b}
$$

Result(type 3, 82 leaves):

$$
\frac{5 \sin (b x+a)}{16 b}-\frac{5 \sin (3 b x+3 a)}{48 b}-\frac{\sin (5 b x+5 a)}{32 b}+\frac{5 \sin (7 b x+7 a)}{224 b}+\frac{\sin (9 b x+9 a)}{288 b}-\frac{\sin (11 b x+11 a)}{352 b}
$$

Problem 5: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a)^{2} \sin (2 b x+2 a)^{3} \mathrm{~d} x
$$

Optimal(type 3, 27 leaves, 4 steps):

$$
\frac{4 \sin (b x+a)^{6}}{3 b}-\frac{\sin (b x+a)^{8}}{b}
$$

Result(type 3, 57 leaves):

$$
-\frac{3 \cos (2 b x+2 a)}{16 b}+\frac{\cos (4 b x+4 a)}{32 b}+\frac{\cos (6 b x+6 a)}{48 b}-\frac{\cos (8 b x+8 a)}{128 b}
$$

Problem 7: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a)^{2} \sin (2 b x+2 a) \mathrm{d} x
$$

Optimal(type 3, 13 leaves, 3 steps):

$$
\frac{\sin (b x+a)^{4}}{2 b}
$$

Result(type 3, 29 leaves):

$$
-\frac{\cos (2 b x+2 a)}{4 b}+\frac{\cos (4 b x+4 a)}{16 b}
$$

Problem 10: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a)^{3} \sin (2 b x+2 a)^{5} \mathrm{~d} x
$$

Optimal(type 3, 40 leaves, 4 steps):

$$
\frac{32 \sin (b x+a)^{9}}{9 b}-\frac{64 \sin (b x+a)^{11}}{11 b}+\frac{32 \sin (b x+a)^{13}}{13 b}
$$

Result(type 3, 96 leaves):

$$
\frac{5 \sin (b x+a)}{32 b}-\frac{25 \sin (3 b x+3 a)}{384 b}-\frac{\sin (5 b x+5 a)}{128 b}+\frac{\sin (7 b x+7 a)}{64 b}-\frac{\sin (9 b x+9 a)}{576 b}-\frac{3 \sin (11 b x+11 a)}{1408 b}+\frac{\sin (13 b x+13 a)}{1664 b}
$$

Problem 24: Attempted integration timed out after 120 seconds.

$$
\int \frac{\sin (b x+a)}{\sin (2 b x+2 a)^{9 / 2}} d x
$$

Optimal(type 3, 89 leaves, 4 steps):

$$
\frac{\sin (b x+a)}{7 b \sin (2 b x+2 a)^{7 / 2}}-\frac{6 \cos (b x+a)}{35 b \sin (2 b x+2 a)^{5 / 2}}+\frac{8 \sin (b x+a)}{35 b \sin (2 b x+2 a)^{3 / 2}}-\frac{16 \cos (b x+a)}{35 b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type 1, 1 leaves):???
Problem 25: Attempted integration timed out after 120 seconds.

$$
\int \sin (b x+a)^{2} \sin (2 b x+2 a)^{7 / 2} \mathrm{~d} x
$$

Optimal(type 4, 109 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{5 \sqrt{\sin \left(a+\frac{\pi}{4}+b x\right)^{2}} \text { EllipticF }\left(\cos \left(a+\frac{\pi}{4}+b x\right), \sqrt{2}\right)}{42 \sin \left(a+\frac{\pi}{4}+b x\right) b}-\frac{\cos (2 b x+2 a) \sin (2 b x+2 a)^{5} / 2}{14 b}-\frac{\sin (2 b x+2 a)^{9} / 2}{18 b} \\
& -\frac{5 \cos (2 b x+2 a) \sqrt{\sin (2 b x+2 a)}}{42 b}
\end{aligned}
$$

Result(type 1, 1 leaves):???

Problem 26: Humongous result has more than 20000 leaves.

$$
\int \frac{\sin (b x+a)^{3}}{\sqrt{\sin (2 b x+2 a)}} d x
$$

Optimal(type 3, 74 leaves, 2 steps):

$$
-\frac{3 \arcsin (\cos (b x+a)-\sin (b x+a))}{8 b}-\frac{3 \ln (\cos (b x+a)+\sin (b x+a)+\sqrt{\sin (2 b x+2 a)})}{8 b}-\frac{\sin (b x+a) \sqrt{\sin (2 b x+2 a)}}{4 b}
$$

Result(type ?, 155738893 leaves): Display of huge result suppressed!
Problem 27: Humongous result has more than 20000 leaves.

$$
\int \frac{\sin (b x+a)^{3}}{\sin (2 b x+2 a)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 73 leaves, 3 steps):

$$
\frac{\arcsin (\cos (b x+a)-\sin (b x+a))}{4 b}-\frac{\ln (\cos (b x+a)+\sin (b x+a)+\sqrt{\sin (2 b x+2 a)})}{4 b}+\frac{\sin (b x+a)}{b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type ?, 149376344 leaves): Display of huge result suppressed!
Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \csc (b x+a) \sqrt{\sin (2 b x+2 a)} \mathrm{d} x
$$

Optimal(type 3, 51 leaves, 2 steps):

$$
-\frac{\arcsin (\cos (b x+a)-\sin (b x+a))}{b}+\frac{\ln (\cos (b x+a)+\sin (b x+a)+\sqrt{\sin (2 b x+2 a)})}{b}
$$

Result(type 4, 156 leaves):
$\left(2 \sqrt{-\frac{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1}}\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}\right.\right.$

$$
-1) \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \operatorname{EllipticF}\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right)
$$

$$
\left(b \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}\right)
$$

Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{\csc (b x+a)}{\sqrt{\sin (2 b x+2 a)}} \mathrm{d} x
$$

Optimal(type 3, 22 leaves, 1 step):

$$
-\frac{\csc (b x+a) \sqrt{\sin (2 b x+2 a)}}{b}
$$

Result (type 4, 307 leaves):
$\frac{1}{b \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}}\left(\sqrt{-\frac{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1}}\left(2 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2}\right.\right.$

$$
\sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \text { EllipticE }\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right)
$$

$$
-\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\left.\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\right)^{2}-1\right)} \operatorname{EllipticF}\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}\right.
$$

$$
\left.\left.\frac{\sqrt{2}}{2}\right)+\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}\right)
$$

Problem 30: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{\csc (b x+a)}{\sin (2 b x+2 a)^{3 / 2}} d x
$$

Optimal(type 3, 45 leaves, 3 steps):

$$
-\frac{2 \cos (b x+a)}{3 b \sin (2 b x+2 a)^{3 / 2}}+\frac{4 \sin (b x+a)}{3 b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type 4, 193 leaves):

$$
\begin{aligned}
& -\left(\sqrt { - \frac { \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) } { \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) ^ { 2 } - 1 } } \left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}\right.\right. \\
& -1)\left(2 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \operatorname{EllipticF}\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)\right. \\
& \left.\left.-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{4}+1\right)\right) /\left(12 b \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}\right)
\end{aligned}
$$

Problem 31: Attempted integration timed out after 120 seconds.

$$
\int \frac{\csc (b x+a)^{2}}{\sin (2 b x+2 a)^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 117 leaves, 4 steps):

$$
\frac{14 \sqrt{\sin \left(a+\frac{\pi}{4}+b x\right)^{2}} \text { EllipticE }\left(\cos \left(a+\frac{\pi}{4}+b x\right), \sqrt{2}\right)}{15 \sin \left(a+\frac{\pi}{4}+b x\right) b}-\frac{14 \cos (2 b x+2 a)}{45 b \sin (2 b x+2 a)^{5 / 2}}-\frac{\csc (b x+a)^{2}}{9 b \sin (2 b x+2 a)^{5 / 2}}-\frac{14 \cos (2 b x+2 a)}{15 b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type 1, 1 leaves):???
Problem 32: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \csc (b x+a)^{3} \sin (2 b x+2 a)^{5 / 2} \mathrm{~d} x
$$

Optimal(type 3, 119 leaves, 5 steps):
$-\frac{3 \arcsin (\cos (b x+a)-\sin (b x+a))}{b}+\frac{3 \ln (\cos (b x+a)+\sin (b x+a)+\sqrt{\sin (2 b x+2 a)})}{b}+\frac{4 \sin (b x+a) \sin (2 b x+2 a)^{3} / 2}{b}$

$$
+\frac{\csc (b x+a)^{3} \sin (2 b x+2 a)^{7 / 2}}{b}-\frac{6 \cos (b x+a) \sqrt{\sin (2 b x+2 a)}}{b}
$$

Result(type 4, 242 leaves):

$$
\begin{aligned}
& 16 \sqrt{-\frac{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1}}\left(\sqrt { \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) + 1 } \sqrt { - 2 \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) + 2 } \sqrt { - \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) } \operatorname { E l l i p t i c F } \left(\sqrt{\left.\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1, \frac{\sqrt{2}}{2}\right) \tan \left(\frac{b x}{2}\right.}\right.\right. \\
& \left.\quad+\frac{a}{2}\right)^{2}-\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \operatorname{EllipticF}\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right)-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3} \\
& \left.\left.\quad-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\right)\right) /\left(3 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} b\right)
\end{aligned}
$$

Problem 33: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \csc (b x+a)^{3} \sin (2 b x+2 a)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 3, 98 leaves, 4 steps):
$\frac{2 \arcsin (\cos (b x+a)-\sin (b x+a))}{b}+\frac{2 \ln (\cos (b x+a)+\sin (b x+a)+\sqrt{\sin (2 b x+2 a)})}{b}-\frac{\csc (b x+a)^{3} \sin (2 b x+2 a)^{5 / 2}}{b}$

Result(type 4, 541 leaves):

$$
\left(4 \sqrt { - \frac { \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) } { \operatorname { t a n } ( \frac { b x } { 2 } + \frac { a } { 2 } ) ^ { 2 } - 1 } } \left(4 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)}\right.\right.
$$

$$
\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1\right)} \operatorname{EllipticE}\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right)
$$

$$
-2 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)}
$$

$$
\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1\right)} \operatorname{EllipticF}\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right)
$$

$$
+\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1\right)} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+2 \tan \left(\frac{b x}{2}\right.
$$

$$
\left.+\frac{a}{2}\right)^{2} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}
$$

$$
\left.\left.-\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1\right)}\right)\right) /\left(\operatorname { t a n } \left(\frac{b x}{2}\right.\right.
$$

$$
\left.\left.+\frac{a}{2}\right) \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1\right)} b\right)
$$

Problem 34: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \csc (b x+a)^{3} \sqrt{\sin (2 b x+2 a)} \mathrm{d} x
$$

Optimal(type 3, 24 leaves, 1 step):

$$
-\frac{\csc (b x+a)^{3} \sin (2 b x+2 a)^{3 / 2}}{3 b}
$$

Result(type 4, 191 leaves):
$\left(\sqrt{-\frac{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1}}\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}\right.\right.$

$$
\begin{aligned}
& -1)\left(4 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \text { EllipticF }\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)\right. \\
& \left.\left.+\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{4}-1\right)\right) /\left(3 \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} b\right)
\end{aligned}
$$

Problem 35: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{\csc (b x+a)^{3}}{\sqrt{\sin (2 b x+2 a)}} d x
$$

Optimal(type 3, 47 leaves, 2 steps):

$$
-\frac{4 \csc (b x+a) \sqrt{\sin (2 b x+2 a)}}{5 b}-\frac{\csc (b x+a)^{3} \sqrt{\sin (2 b x+2 a)}}{5 b}
$$

Result(type 4, 481 leaves):

$$
\frac{1}{20 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} b} \sqrt{-\frac{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1}}\left(16 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)}\right.
$$

$$
\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \text { EllipticE }\left(\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}
$$

$$
-8 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1} \sqrt{-2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2} \sqrt{-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \text { EllipticF }
$$

$$
\left.\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1}, \frac{\sqrt{2}}{2}\right) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{6}+\tan \left(\frac{b x}{2}\right.
$$

$$
\left.+\frac{a}{2}\right)^{4} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)}+8 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{4} \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)}
$$

$$
+\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-8 \sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-\tan \left(\frac{b x}{2}+\frac{a}{2}\right)} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}
$$

$$
\left.\left.-\sqrt{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)}\right)\right)
$$

[^0]$$
\int \frac{\csc (b x+a)^{3}}{\sin (2 b x+2 a)^{5 / 2}} d x
$$

Optimal(type 3, 91 leaves, 5 steps):

$$
-\frac{8 \cos (b x+a)}{15 b \sin (2 b x+2 a)^{5 / 2}}-\frac{\csc (b x+a)^{3}}{9 b \sin (2 b x+2 a)^{3 / 2}}+\frac{32 \sin (b x+a)}{45 b \sin (2 b x+2 a)^{3 / 2}}-\frac{64 \cos (b x+a)}{45 b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type 1, 1 leaves):???
Problem 37: Unable to integrate problem.

$$
\int \sin (b x+a)^{2} \sin (2 b x+2 a)^{m} \mathrm{~d} x
$$

Optimal(type 5, 74 leaves, 2 steps):

$$
\frac{\left(\cos (b x+a)^{2}\right)^{\frac{1}{2}-\frac{m}{2}} \text { hypergeom }\left(\left[\frac{1}{2}-\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right],\left[\frac{5}{2}+\frac{m}{2}\right], \sin (b x+a)^{2}\right) \sin (b x+a)^{2} \sin (2 b x+2 a)^{m} \tan (b x+a)}{b(3+m)}
$$

Result(type 8, 22 leaves):

$$
\int \sin (b x+a)^{2} \sin (2 b x+2 a)^{m} \mathrm{~d} x
$$

Problem 38: Unable to integrate problem.

$$
\int \csc (b x+a) \sin (2 b x+2 a)^{m} \mathrm{~d} x
$$

Optimal(type 5, 62 leaves, 2 steps):

$$
\frac{\left(\cos (b x+a)^{2}\right)^{\frac{1}{2}-\frac{m}{2}} \text { hypergeom }\left(\left[\frac{m}{2}, \frac{1}{2}-\frac{m}{2}\right],\left[1+\frac{m}{2}\right], \sin (b x+a)^{2}\right) \sec (b x+a) \sin (2 b x+2 a)^{m}}{b m}
$$

Result(type 8, 20 leaves):

$$
\int \csc (b x+a) \sin (2 b x+2 a)^{m} \mathrm{~d} x
$$

Problem 39: Result more than twice size of optimal antiderivative.

$$
\int \cos (b x+a) \sin (2 b x+2 a)^{5} \mathrm{~d} x
$$

Optimal(type 3, 40 leaves, 4 steps):

$$
-\frac{32 \cos (b x+a)^{7}}{7 b}+\frac{64 \cos (b x+a)^{9}}{9 b}-\frac{32 \cos (b x+a)^{11}}{11 b}
$$

Result(type 3, 82 leaves):

$$
-\frac{5 \cos (b x+a)}{16 b}-\frac{5 \cos (3 b x+3 a)}{48 b}+\frac{\cos (5 b x+5 a)}{32 b}+\frac{5 \cos (7 b x+7 a)}{224 b}-\frac{\cos (9 b x+9 a)}{288 b}-\frac{\cos (11 b x+11 a)}{352 b}
$$

Problem 45: Result more than twice size of optimal antiderivative.

$$
\int \cos (b x+a)^{3} \sin (2 b x+2 a) d x
$$

Optimal (type 3, 13 leaves, 3 steps):

$$
-\frac{2 \cos (b x+a)^{5}}{5 b}
$$

Result (type 3, 40 leaves):

$$
-\frac{\cos (b x+a)}{4 b}-\frac{\cos (3 b x+3 a)}{8 b}-\frac{\cos (5 b x+5 a)}{40 b}
$$

Problem 47: Humongous result has more than 20000 leaves.

$$
\int \frac{\cos (b x+a)}{\sin (2 b x+2 a)^{3 / 2}} d x
$$

Optimal(type 3, 22 leaves, 1 step):

$$
-\frac{\cos (b x+a)}{b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type ?, 55916573 leaves): Display of huge result suppressed!
Problem 48: Attempted integration timed out after 120 seconds.

$$
\int \frac{\cos (b x+a)}{\sin (2 b x+2 a)^{7 / 2}} d x
$$

Optimal(type 3, 67 leaves, 3 steps):

$$
-\frac{\cos (b x+a)}{5 b \sin (2 b x+2 a)^{5 / 2}}+\frac{4 \sin (b x+a)}{15 b \sin (2 b x+2 a)^{3 / 2}}-\frac{8 \cos (b x+a)}{15 b \sqrt{\sin (2 b x+2 a)}}
$$

Result(type 1, 1 leaves):???
Problem 49: Unable to integrate problem.

$$
\int \cos (b x+a)^{2} \sin (2 b x+2 a)^{m} \mathrm{~d} x
$$

Optimal(type 5, 75 leaves, 2 steps):

$$
-\frac{\cos (b x+a)^{2} \cot (b x+a) \text { hypergeom }\left(\left[\frac{1}{2}-\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right],\left[\frac{5}{2}+\frac{m}{2}\right], \cos (b x+a)^{2}\right)\left(\sin (b x+a)^{2}\right)^{\frac{1}{2}-\frac{m}{2}} \sin (2 b x+2 a)^{m}}{b(3+m)}
$$

Result (type 8, 22 leaves):

$$
\int \cos (b x+a)^{2} \sin (2 b x+2 a)^{m} \mathrm{~d} x
$$

Problem 51: Result more than twice size of optimal antiderivative.

$$
\int \csc (b x+c)^{3} \sin (b x+a) \mathrm{d} x
$$

Optimal(type 3, 37 leaves, 5 steps):

$$
-\frac{\cos (a-c) \cot (b x+c)}{b}-\frac{\csc (b x+c)^{2} \sin (a-c)}{2 b}
$$

Result(type 3, 119 leaves):
$\frac{1}{b}\left(-\frac{1}{(\cos (a) \cos (c)+\sin (a) \sin (c))^{2}(\tan (b x+a) \cos (a) \cos (c)+\tan (b x+a) \sin (a) \sin (c)+\cos (a) \sin (c)-\sin (a) \cos (c))}\right.$
$\left.-\frac{\sin (a) \cos (c)-\cos (a) \sin (c)}{2(\cos (a) \cos (c)+\sin (a) \sin (c))^{2}(\tan (b x+a) \cos (a) \cos (c)+\tan (b x+a) \sin (a) \sin (c)+\cos (a) \sin (c)-\sin (a) \cos (c))^{2}}\right)$

Problem 52: Humongous result has more than 20000 leaves.

$$
\int \csc (b x+c)^{6} \sin (b x+a) d x
$$

Optimal(type 3, 86 leaves, 6 steps):
$-\frac{3 \operatorname{arctanh}(\cos (b x+c)) \cos (a-c)}{8 b}-\frac{3 \cos (a-c) \cot (b x+c) \csc (b x+c)}{8 b}-\frac{\cos (a-c) \cot (b x+c) \csc (b x+c)^{3}}{4 b}-\frac{\csc (b x+c)^{5} \sin (a-c)}{5 b}$
Result(type ?, 97947 leaves): Display of huge result suppressed!
Problem 53: Unable to integrate problem.

$$
\int \sin (b x+a)^{3} \sin (d x+c)^{n} \mathrm{~d} x
$$

Optimal(type 5, 540 leaves, 18 steps):

$$
\begin{aligned}
& 2^{-3-n} \mathrm{e}^{\mathrm{I}(-c n+3 a)+\mathrm{I}(-n d+3 b) x+\mathrm{I} n(d x+c)}\left(\frac{\mathrm{I}}{\mathrm{e}^{\mathrm{I}(d x+c)}}-\mathrm{I} \mathrm{e}^{\mathrm{I}(d x+c)}\right)^{n} \operatorname{hypergeom}\left(\left[-n, \frac{3 b}{2 d}-\frac{n}{2}\right],\left[1+\frac{3 b}{2 d}-\frac{n}{2}\right], \mathrm{e}^{2 \mathrm{I}(d x+c)}\right) \\
&-\frac{\left(1-\mathrm{e}^{2 \mathrm{I} c+2 \mathrm{I} d x}\right)^{n}(-n d+3 b)}{32^{-3-n} \mathrm{e}^{\mathrm{I}(-c n+a)+\mathrm{I}(-n d+b) x+\mathrm{I} n(d x+c)}\left(\frac{\mathrm{I}}{\mathrm{e}^{\mathrm{I}(d x+c)}}-\mathrm{I}^{\mathrm{I}(d x+c)}\right)^{n} \operatorname{hypergeom}\left(\left[-n, \frac{-n d+b}{2 d}\right],\left[1+\frac{b}{2 d}-\frac{n}{2}\right], \mathrm{e}^{2 \mathrm{I}(d x+c)}\right)} \\
&-\frac{\left(1-\mathrm{e}^{2 \mathrm{I} c+2 \mathrm{I} d x}\right)^{n}(-n d+b)}{} \\
&+\frac{2^{-3-n} 2^{-3-n} \mathrm{e}^{-\mathrm{I}(c n+a)-\mathrm{I}(n d+b) x+\mathrm{I} n(d x+c)}\left(\frac{\mathrm{I}}{\mathrm{e}^{\mathrm{I}(d x+c)}}-\mathrm{I} \mathrm{e}^{\mathrm{I}(d x+c)}\right)^{n} \operatorname{hypergeom}\left(\left[-n, \frac{-n d-b}{2 d}\right],\left[1+\frac{-n d-b}{2 d}\right], \mathrm{e}^{2 \mathrm{I}(d x+c)}\right)}{(1-\mathrm{I}(n d+3 b) x+\mathrm{I} n(d x+c)}\left(\frac{\mathrm{I}}{\left.\mathrm{e}^{2 \mathrm{I}(d x+2 \mathrm{I} d x}\right)^{n}(n d+b)}-\mathrm{I} \mathrm{e}^{\mathrm{I}(d x+c)}\right)^{n} \operatorname{hypergeom}\left(\left[-n, \frac{-n d-3 b}{2 d}\right],\left[1-\frac{3 b}{2 d}-\frac{n}{2}\right], \mathrm{e}^{2 \mathrm{I}(d x+c)}\right) \\
&\left(1-\mathrm{e}^{2 \mathrm{I} c+2 \mathrm{I} d x}\right)^{n}(n d+3 b)
\end{aligned}
$$

Result(type 8, 19 leaves):

$$
\int \sin (b x+a)^{3} \sin (d x+c)^{n} \mathrm{~d} x
$$

Problem 56: Result more than twice size of optimal antiderivative.

$$
\int \sec (b x+c)^{5} \sin (b x+a) \mathrm{d} x
$$

Optimal(type 3, 55 leaves, 5 steps):

$$
\frac{\cos (a-c) \sec (b x+c)^{4}}{4 b}+\frac{\sin (a-c) \tan (b x+c)}{b}+\frac{\sin (a-c) \tan (b x+c)^{3}}{3 b}
$$

Result(type 3, 380 leaves):
$\frac{1}{b}\left(-\left(-3 \cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-4 \cos (a) \sin (a) \cos (c) \sin (c)-\sin (a)^{2} \cos (c)^{2}-3 \sin (a)^{2} \sin (c)^{2}\right) /(3(\cos (a) \sin (c)\right.$
$-\sin (a) \cos (c))^{3}(\sin (a) \cos (c)-\cos (a) \sin (c))(-\tan (b x+a) \cos (a) \sin (c)+\tan (b x+a) \sin (a) \cos (c)+\cos (a) \cos (c)$
$\left.+\sin (a) \sin (c))^{3}\right)-\left((\cos (a) \cos (c)+\sin (a) \sin (c))\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)\right) /$
$\left(4(\cos (a) \sin (c)-\sin (a) \cos (c))^{3}(\sin (a) \cos (c)-\cos (a) \sin (c))(-\tan (b x+a) \cos (a) \sin (c)+\tan (b x+a) \sin (a) \cos (c)+\cos (a) \cos (c)\right.$
 $\left.+\sin (a) \sin (c))^{2}\right)+1 /\left((\cos (a) \sin (c)-\sin (a) \cos (c))^{3}(\sin (a) \cos (c)-\cos (a) \sin (c))(-\tan (b x+a) \cos (a) \sin (c)+\tan (b x\right.$ $+a) \sin (a) \cos (c)+\cos (a) \cos (c)+\sin (a) \sin (c))))$

Problem 60: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (b x+a)}{\sin (b x+c)^{2}} d x
$$

Optimal(type 3, 35 leaves, 4 steps):

$$
-\frac{\cos (a-c) \csc (b x+c)}{b}+\frac{\operatorname{arctanh}(\cos (b x+c)) \sin (a-c)}{b}
$$

Result (type 3, 1055 leaves):
$-\left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a)^{2} \cos (c)^{2}\right) /\left(b\left(-\frac{\cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\frac{\sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a) \cos (c)\right.\right.$
$\left.+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sin (a) \sin (c)+\frac{\cos (a) \sin (c)}{2}-\frac{\sin (a) \cos (c)}{2}\right)\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}\right.$
$\left.+\sin (a)^{2} \sin (c)^{2}\right)(\cos (a) \sin (c)-\sin (a) \cos (c))-\left(2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a) \sin (a) \cos (c) \sin (c)\right) /(b)$
$-\frac{\cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\frac{\sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a) \cos (c)+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sin (a) \sin (c)+\frac{\cos (a) \sin (c)}{2}$
$\left.\left.-\frac{\sin (a) \cos (c)}{2}\right)\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)(\cos (a) \sin (c)-\sin (a) \cos (c))\right)-\left(\tan \left(\frac{b x}{2}\right.\right.$
$\left.\left.+\frac{a}{2}\right) \sin (a)^{2} \sin (c)^{2}\right) /\left(b\left(-\frac{\cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\frac{\sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a) \cos (c)+\tan \left(\frac{b x}{2}\right.\right.\right.$
$\left.\left.+\frac{a}{2}\right) \sin (a) \sin (c)+\frac{\cos (a) \sin (c)}{2}-\frac{\sin (a) \cos (c)}{2}\right)\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)(\cos (a) \sin (c)$
$-\sin (a) \cos (c))-(\cos (a) \cos (c)) /\left(b\left(-\frac{\cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\frac{\sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\tan \left(\frac{b x}{2}\right.\right.\right.$
$\left.\left.+\frac{a}{2}\right) \cos (a) \cos (c)+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sin (a) \sin (c)+\frac{\cos (a) \sin (c)}{2}-\frac{\sin (a) \cos (c)}{2}\right)\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}\right.$
$\left.\left.+\sin (a)^{2} \sin (c)^{2}\right)\right)-(\sin (a) \sin (c)) /\left(b\left(-\frac{\cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\frac{\sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{2}+\tan \left(\frac{b x}{2}\right.\right.\right.$
$\left.\left.+\frac{a}{2}\right) \cos (a) \cos (c)+\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sin (a) \sin (c)+\frac{\cos (a) \sin (c)}{2}-\frac{\sin (a) \cos (c)}{2}\right)\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}\right.$
$\left.\left.+\sin (a)^{2} \sin (c)^{2}\right)\right)$
$+\left(2 \arctan \left(\frac{2(-2 \cos (a) \sin (c)+2 \sin (a) \cos (c)) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+4 \cos (a) \cos (c)+4 \sin (a) \sin (c)}{4 \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}}\right) \cos (a) \sin (c)\right) /$
$\left(b\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}\right.\right.$
$\left.\left.+\sin (a)^{2} \sin (c)^{2}\right) \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}\right)$
$-\left(2 \arctan \left(\frac{2(-2 \cos (a) \sin (c)+2 \sin (a) \cos (c)) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+4 \cos (a) \cos (c)+4 \sin (a) \sin (c)}{4 \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}}\right) \sin (a) \cos (c)\right) /$
$\left(b\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}\right.\right.$
$\left.\left.+\sin (a)^{2} \sin (c)^{2}\right) \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}\right)$

Problem 61: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a) \tan (b x+c)^{3} \mathrm{~d} x
$$

Optimal (type 3, 68 leaves, 9 steps):

$$
-\frac{3 \operatorname{arctanh}(\sin (b x+c)) \cos (a-c)}{2 b}+\frac{\sec (b x+c) \sin (a-c)}{b}+\frac{\sin (b x+a)}{b}+\frac{\cos (a-c) \sec (b x+c) \tan (b x+c)}{2 b}
$$

Result(type 3, 185 leaves):

$$
\begin{aligned}
& -\frac{\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}}{2 b}+\frac{\mathrm{I} \mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b}-\frac{\mathrm{I}\left(3 \mathrm{e}^{\mathrm{I}(3 b x+5 a+2 c)}-\mathrm{e}^{\mathrm{I}(3 b x+3 a+4 c)}+\mathrm{e}^{\mathrm{I}(b x+5 a)}-3 \mathrm{e}^{\mathrm{I}(b x+3 a+2 c)}\right)}{2 b\left(\mathrm{e}^{2 \mathrm{I}(b x+a+c)}+\mathrm{e}^{2 \mathrm{I} a}\right)^{2}}+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-\mathrm{I} \mathrm{e}^{\mathrm{I}(a-c)}\right) \cos (a-c)}{2 b} \\
& \quad-\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+\mathrm{I} \mathrm{e}^{\mathrm{I}(a-c)}\right) \cos (a-c)}{2 b}
\end{aligned}
$$

Problem 62: Result more than twice size of optimal antiderivative.

$$
\int \sin (b x+a) \tan (b x+c)^{2} \mathrm{~d} x
$$

Optimal(type 3, 44 leaves, 6 steps):

$$
\frac{\cos (b x+a)}{b}+\frac{\cos (a-c) \sec (b x+c)}{b}+\frac{\operatorname{arctanh}(\sin (b x+c)) \sin (a-c)}{b}
$$

Result(type 3, 142 leaves):

$$
\frac{\mathrm{e}^{\mathrm{I}(b x+a)}}{2 b}+\frac{\mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b}+\frac{\mathrm{e}^{\mathrm{I}(b x+3 a)}+\mathrm{e}^{\mathrm{I}(b x+a+2 c)}}{b\left(\mathrm{e}^{2 \mathrm{I}(b x+a+c)}+\mathrm{e}^{2 \mathrm{I} a}\right)}+\frac{\ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+\mathrm{I}^{\mathrm{I}(a-c)}\right) \sin (a-c)}{b}-\frac{\ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-\mathrm{I} \mathrm{e}^{\mathrm{I}(a-c)}\right) \sin (a-c)}{b}
$$

Problem 63: Result more than twice size of optimal antiderivative.

$$
\int \cot (b x+c) \sin (b x+a) \mathrm{d} x
$$

Optimal (type 3, 29 leaves, 3 steps):

$$
-\frac{\operatorname{arctanh}(\cos (b x+c)) \sin (a-c)}{b}+\frac{\sin (b x+a)}{b}
$$

Result(type 3, 94 leaves):

$$
-\frac{\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}}{2 b}+\frac{\mathrm{I} \mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b}+\frac{\ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-\mathrm{e}^{\mathrm{I}(a-c)}\right) \sin (a-c)}{b}-\frac{\ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+\mathrm{e}^{\mathrm{I}(a-c)}\right) \sin (a-c)}{b}
$$

Problem 66: Result more than twice size of optimal antiderivative.

$$
\int \cos (b x+a) \sec (b x+c)^{2} d x
$$

Optimal(type 3, 35 leaves, 4 steps):

$$
\frac{\operatorname{arctanh}(\sin (b x+c)) \cos (a-c)}{b}-\frac{\sec (b x+c) \sin (a-c)}{b}
$$

Result(type 3, 1048 leaves):
$\left(2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a)^{2} \sin (c)^{2}\right) /\left(b\left(\cos (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+\sin (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+2 \cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)\right.\right.$
$\left.-2 \sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)-\cos (a) \cos (c)-\sin (a) \sin (c)\right)(\cos (a) \cos (c)+\sin (a) \sin (c))\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}\right.$
$\left.\left.+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)\right)-\left(4 \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \cos (a) \sin (a) \cos (c) \sin (c)\right) /\left(b\left(\cos (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}\right.\right.$ $\left.+\sin (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+2 \cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)-2 \sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)-\cos (a) \cos (c)-\sin (a) \sin (c)\right)(\cos (a) \cos (c)$ $\left.+\sin (a) \sin (c))\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)\right)+\left(2 \tan \left(\frac{b x}{2}+\frac{a}{2}\right) \sin (a)^{2} \cos (c)^{2}\right) /$ $\left(b\left(\cos (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+\sin (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+2 \cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)-2 \sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)\right.\right.$ $\left.-\cos (a) \cos (c)-\sin (a) \sin (c))(\cos (a) \cos (c)+\sin (a) \sin (c))\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)\right)$ $-(2 \cos (a) \sin (c)) /\left(b\left(\cos (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+\sin (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+2 \cos (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)-2 \sin (a) \cos (c) \tan \left(\frac{b x}{2}+\right.\right.\right.$ $\left.\left.+\sin (a)^{2} \sin (c)^{2}\right)\right)+(2 \sin (a) \cos (c)) /\left(b\left(\cos (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+\sin (a) \sin (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}+2 \cos (a) \sin (c) \tan \left(\frac{b x}{2}\right.\right.\right.$ $\left.\left.\left.+\frac{a}{2}\right)-2 \sin (a) \cos (c) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)-\cos (a) \cos (c)-\sin (a) \sin (c)\right)\left(\cos (a)^{2} \cos (c)^{2}+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right)\right)$
$-\left(2 \arctan \left(\frac{2(\cos (a) \cos (c)+\sin (a) \sin (c)) \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+2 \cos (a) \sin (c)-2 \sin (a) \cos (c)}{2 \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}}\right) \cos (a) \cos (c)\right) /\left(b\left(\cos (a)^{2} \cos (c)^{2}\right.\right.$
$\left.\left.+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right) \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}\right)$
 $\left.\left.+\cos (a)^{2} \sin (c)^{2}+\sin (a)^{2} \cos (c)^{2}+\sin (a)^{2} \sin (c)^{2}\right) \sqrt{-\cos (a)^{2} \cos (c)^{2}-\cos (a)^{2} \sin (c)^{2}-\sin (a)^{2} \cos (c)^{2}-\sin (a)^{2} \sin (c)^{2}}\right)$

Problem 68: Result more than twice size of optimal antiderivative.

$$
\int \cos (b x+a) \tan (b x+c)^{3} \mathrm{~d} x
$$

Optimal(type 3, 68 leaves, 9 steps):

$$
\frac{\cos (b x+a)}{b}+\frac{\cos (a-c) \sec (b x+c)}{b}+\frac{3 \operatorname{arctanh}(\sin (b x+c)) \sin (a-c)}{2 b}-\frac{\sec (b x+c) \sin (a-c) \tan (b x+c)}{2 b}
$$

[^1]\[

$$
\begin{aligned}
& \frac{\mathrm{e}^{\mathrm{I}(b x+a)}}{2 b}+\frac{\mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b}+\frac{3 \mathrm{e}^{\mathrm{I}(3 b x+5 a+2 c)}+\mathrm{e}^{\mathrm{I}(3 b x+3 a+4 c)}+\mathrm{e}^{\mathrm{I}(b x+5 a)}+3 \mathrm{e}^{\mathrm{I}(b x+3 a+2 c)}}{2 b\left(\mathrm{e}^{2 \mathrm{I}(b x+a+c)}+\mathrm{e}^{2 \mathrm{I} a}\right)^{2}}-\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-\mathrm{I} \mathrm{e}^{\mathrm{I}(a-c)}\right) \sin (a-c)}{2 b} \\
& \quad+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+\mathrm{I}^{\mathrm{I}(a-c)}\right) \sin (a-c)}{2 b}
\end{aligned}
$$
\]

Problem 69: Result more than twice size of optimal antiderivative.

$$
\int \cos (b x+a) \cot (b x+c) \mathrm{d} x
$$

Optimal(type 3, 29 leaves, 3 steps):

$$
-\frac{\operatorname{arctanh}(\cos (b x+c)) \cos (a-c)}{b}+\frac{\cos (b x+a)}{b}
$$

Result(type 3, 92 leaves):

$$
\frac{\mathrm{e}^{\mathrm{I}(b x+a)}}{2 b}+\frac{\mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b}-\frac{\ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+\mathrm{e}^{\mathrm{I}(a-c)}\right) \cos (a-c)}{b}+\frac{\ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-\mathrm{e}^{\mathrm{I}(a-c)}\right) \cos (a-c)}{b}
$$

Test results for the 78 problems in "4.7.2 trig^m (a trig+b trig)^n.txt"
Problem 10: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sin (x)^{2}}{(a \cos (x)+b \sin (x))^{3}} \mathrm{~d} x
$$

Optimal(type 3, 84 leaves, ? steps):

$$
-\frac{\left(a^{2}-2 b^{2}\right) \operatorname{arctanh}\left(\frac{-b+a \tan \left(\frac{x}{2}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{5 / 2}}+\frac{a\left(3 a b \cos (x)+\left(a^{2}+4 b^{2}\right) \sin (x)\right)}{2\left(a^{2}+b^{2}\right)^{2}(a \cos (x)+b \sin (x))^{2}}
$$

Result(type 3, 211 leaves):

$$
\begin{aligned}
&-\frac{8\left(-\frac{a\left(a^{2}-2 b^{2}\right) \tan \left(\frac{x}{2}\right)^{3}}{8\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{3 b\left(a^{2}-2 b^{2}\right) \tan \left(\frac{x}{2}\right)^{2}}{8\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{\left(a^{2}+10 b^{2}\right) a \tan \left(\frac{x}{2}\right)}{8\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{3 a^{2} b}{8\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}\right)}{\left(\tan \left(\frac{x}{2}\right)^{2} a-2 \tan \left(\frac{x}{2}\right) b-a\right)^{2}} \\
&-\frac{\left(a^{2}-2 b^{2}\right) \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{x}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) \sqrt{a^{2}+b^{2}}}
\end{aligned}
$$

Problem 11: Unable to integrate problem.

$$
\int \frac{(a \cos (d x+c)+\mathrm{I} a \sin (d x+c))^{n}}{\sin (d x+c)^{n}} \mathrm{~d} x
$$

Optimal(type 5, 60 leaves, 1 step):

$$
\frac{-\frac{\mathrm{I}}{2} \text { hypergeom }\left([1, n],[1+n],-\frac{\mathrm{I}}{2}(\mathrm{I}+\cot (d x+c))\right)(a \cos (d x+c)+\mathrm{I} a \sin (d x+c))^{n}}{d n \sin (d x+c)^{n}}
$$

Result(type 8, 34 leaves):

$$
\int \frac{(a \cos (d x+c)+\mathrm{I} a \sin (d x+c))^{n}}{\sin (d x+c)^{n}} \mathrm{~d} x
$$

Problem 32: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (d x+c)^{2}}{a \cos (d x+c)+b \sin (d x+c)} \mathrm{d} x
$$

Optimal(type 3, 76 leaves, 4 steps):

$$
-\frac{a \operatorname{arctanh}(\sin (d x+c))}{b^{2} d}+\frac{\sec (d x+c)}{b d}-\frac{\operatorname{arctanh}\left(\frac{b \cos (d x+c)-a \sin (d x+c)}{\sqrt{a^{2}+b^{2}}}\right) \sqrt{a^{2}+b^{2}}}{b^{2} d}
$$

Result(type 3, 173 leaves):

$$
\begin{gathered}
-\frac{1}{d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{a \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{d b^{2}}+\frac{1}{d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}-\frac{a \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{d b^{2}} \\
+\frac{2 \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right) a^{2}}{d b^{2} \sqrt{a^{2}+b^{2}}}+\frac{2 \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{d \sqrt{a^{2}+b^{2}}}
\end{gathered}
$$

Problem 34: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (d x+c)^{4}}{a \cos (d x+c)+b \sin (d x+c)} \mathrm{d} x
$$

Optimal(type 3, 143 leaves, 7 steps):
$-\frac{a \operatorname{arctanh}(\sin (d x+c))}{2 b^{2} d}-\frac{a\left(a^{2}+b^{2}\right) \operatorname{arctanh}(\sin (d x+c))}{b^{4} d}-\frac{\left(a^{2}+b^{2}\right)^{3 / 2} \operatorname{arctanh}\left(\frac{b \cos (d x+c)-a \sin (d x+c)}{\sqrt{a^{2}+b^{2}}}\right)}{b^{4} d}+\frac{\left(a^{2}+b^{2}\right) \sec (d x+c)}{b^{3} d}$

$$
+\frac{\sec (d x+c)^{3}}{3 b d}-\frac{a \sec (d x+c) \tan (d x+c)}{2 b^{2} d}
$$

Result(type 3, 487 leaves):

$$
\begin{aligned}
& -\frac{1}{3 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a}{2 d b^{2}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{2 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a^{2}}{d b^{3}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)} \\
& -\frac{a}{2 d b^{2}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}-\frac{3}{2 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{a^{3} \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{d b^{4}}+\frac{3 a \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2 d b^{2}} \\
& +\frac{1}{3 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{a}{2 d b^{2}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{a^{2}}{d b^{3}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)} \\
& -\frac{a}{2 d b^{2}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}+\frac{3}{2 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}-\frac{a^{3} \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{d b^{4}}-\frac{3 a \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{2 d b^{2}} \\
& +\frac{2 \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right) a^{4}}{d b^{4} \sqrt{a^{2}+b^{2}}}+\frac{4 \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right) a^{2}}{d b^{2} \sqrt{a^{2}+b^{2}}}+\frac{2 \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{d \sqrt{a^{2}+b^{2}}}
\end{aligned}
$$

Problem 36: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (d x+c)^{4}}{(a \cos (d x+c)+b \sin (d x+c))^{2}} d x
$$

Optimal(type 3, 141 leaves, 7 steps):

$$
\begin{aligned}
& \frac{\left(a^{4}+6 a^{2} b^{2}-3 b^{4}\right) x}{2\left(a^{2}+b^{2}\right)^{3}}+\frac{b^{4}}{a\left(a^{2}+b^{2}\right)^{2} d(b+a \cot (d x+c))}+\frac{4 a b^{3} \ln (a \cos (d x+c)+b \sin (d x+c))}{\left(a^{2}+b^{2}\right)^{3} d} \\
& -\frac{\left(2 a b-\left(a^{2}-b^{2}\right) \cot (d x+c)\right) \sin (d x+c)^{2}}{2\left(a^{2}+b^{2}\right)^{2} d}
\end{aligned}
$$

Result(type 3, 291 leaves):

$$
\begin{aligned}
\frac{\tan (d x+c) a^{4}}{2 d\left(a^{2}+b^{2}\right)^{3}\left(\tan (d x+c)^{2}+1\right)} & -\frac{\tan (d x+c) b^{4}}{2 d\left(a^{2}+b^{2}\right)^{3}\left(\tan (d x+c)^{2}+1\right)}+\frac{a^{3} b}{d\left(a^{2}+b^{2}\right)^{3}\left(\tan (d x+c)^{2}+1\right)}+\frac{a b^{3}}{d\left(a^{2}+b^{2}\right)^{3}\left(\tan (d x+c)^{2}+1\right)} \\
-\frac{2 a b^{3} \ln \left(\tan (d x+c)^{2}+1\right)}{d\left(a^{2}+b^{2}\right)^{3}} & +\frac{3 \arctan (\tan (d x+c)) a^{2} b^{2}}{d\left(a^{2}+b^{2}\right)^{3}}-\frac{3 \arctan (\tan (d x+c)) b^{4}}{2 d\left(a^{2}+b^{2}\right)^{3}}+\frac{\arctan (\tan (d x+c)) a^{4}}{2 d\left(a^{2}+b^{2}\right)^{3}}
\end{aligned}
$$

$$
-\frac{b^{3}}{d\left(a^{2}+b^{2}\right)^{2}(a+b \tan (d x+c))}+\frac{4 b^{3} a \ln (a+b \tan (d x+c))}{d\left(a^{2}+b^{2}\right)^{3}}
$$

Problem 40: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (d x+c)^{2}}{(a \cos (d x+c)+b \sin (d x+c))^{3}} \mathrm{~d} x
$$

Optimal(type 3, 112 leaves, ? steps):

$$
\frac{\left(2 a^{2}-b^{2}\right) \operatorname{arctanh}\left(\frac{-b+a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{5 / 2} d}-\frac{b\left(\left(4 a^{2}+b^{2}\right) \cos (d x+c)+3 a b \sin (d x+c)\right)}{2\left(a^{2}+b^{2}\right)^{2} d(a \cos (d x+c)+b \sin (d x+c))^{2}}
$$

Result(type 3, 279 leaves):

$$
\begin{aligned}
& \frac{1}{d}\left(-\frac{2\left(-\frac{b^{2}\left(5 a^{2}+2 b^{2}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) a}-\frac{b\left(4 a^{4}-7 a^{2} b^{2}-2 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) a^{2}}+\frac{b^{2}\left(11 a^{2}+2 b^{2}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) a}+\frac{b\left(4 a^{2}+b^{2}\right)}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{2}}\right. \\
& \left.+\frac{\left.\left(2 a^{2}-b^{2}\right) \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)\right)}{\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) \sqrt{a^{2}+b^{2}}}\right)
\end{aligned}
$$

Problem 41: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (d x+c)^{3}}{(a \cos (d x+c)+b \sin (d x+c))^{4}} \mathrm{~d} x
$$

Optimal(type 3, 151 leaves, ? steps):

$$
\frac{a\left(2 a^{2}-3 b^{2}\right) \operatorname{arctanh}\left(\frac{-b+a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{7 / 2} d}+\frac{-3\left(3 a^{4} b-a^{2} b^{3}+b^{5}\right) \cos (2 d x+2 c)+\frac{b\left(-9 a^{2}+b^{2}\right)\left(2 a^{2}+2 b^{2}+3 a b \sin (2 d x+2 c)\right)}{2}}{6\left(a^{2}+b^{2}\right)^{3} d(a \cos (d x+c)+b \sin (d x+c))^{3}}
$$

Result(type 3, 493 leaves):

$$
\begin{aligned}
& \frac{1}{d}\left(-\frac{1}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}\left(2 \left(-\frac{b^{2}\left(9 a^{4}+6 a^{2} b^{2}+2 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{2 a\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)}\right.\right.\right. \\
&-\frac{b\left(6 a^{6}-27 a^{4} b^{2}-12 a^{2} b^{4}-4 b^{6}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{2 a^{2}\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)}+\frac{b^{2}\left(54 a^{6}-21 a^{4} b^{2}-4 a^{2} b^{4}-4 b^{6}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 a^{3}\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)} \\
&\left.\left.+\frac{b\left(6 a^{6}-20 a^{4} b^{2}-3 a^{2} b^{4}-2 b^{6}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{a^{2}\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)}-\frac{b^{2}\left(27 a^{4}+4 a^{2} b^{2}+2 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{2 a\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)}-\frac{b\left(18 a^{4}+5 a^{2} b^{2}+2 b^{4}\right)}{6\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)}\right)\right) \\
&\left.+\frac{\left.a\left(2 a^{2}-3 b^{2}\right) \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)\right)}{\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right) \sqrt{a^{2}+b^{2}}}\right)
\end{aligned}
$$

Problem 43: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (d x+c)^{3}}{(a \cos (d x+c)+b \sin (d x+c))^{4}} \mathrm{~d} x
$$

Optimal(type 3, 382 leaves, 32 steps):
$\frac{8 a^{2} \operatorname{arctanh}(\sin (d x+c))}{b^{6} d}+\frac{\operatorname{arctanh}(\sin (d x+c))}{2 b^{4} d}+\frac{2\left(a^{2}+b^{2}\right) \operatorname{arctanh}(\sin (d x+c))}{b^{6} d}-\frac{4 a \sec (d x+c)}{b^{5} d}+\frac{-a^{2}-b^{2}}{3 b^{3} d(a \cos (d x+c)+b \sin (d x+c))^{3}}$
$+\frac{3 a(b \cos (d x+c)-a \sin (d x+c))}{2 b^{4} d(a \cos (d x+c)+b \sin (d x+c))^{2}}-\frac{4 a^{2}}{b^{5} d(a \cos (d x+c)+b \sin (d x+c))}-\frac{2\left(a^{2}+b^{2}\right)}{b^{5} d(a \cos (d x+c)+b \sin (d x+c))}$
$\begin{aligned} & \frac{4 a^{3} \operatorname{arctanh}\left(\frac{b \cos (d x+c)-a \sin (d x+c)}{\sqrt{a^{2}+b^{2}}}\right)}{b^{6} d \sqrt{a^{2}+b^{2}}}+\frac{3 a \operatorname{arctanh}\left(\frac{b \cos (d x+c)-a \sin (d x}{\sqrt{a^{2}+b^{2}}}\right.}{2 b^{4} d \sqrt{a^{2}+b^{2}}} \\ + & \frac{6 a \operatorname{arctanh}\left(\frac{b \cos (d x+c)-a \sin (d x+c)}{\sqrt{a^{2}+b^{2}}}\right) \sqrt{a^{2}+b^{2}}}{b^{6} d}+\frac{\sec (d x+c) \tan (d x+c)}{2 b^{4} d}\end{aligned}$
Result(type 3, 1254 leaves):

$$
\begin{aligned}
& \frac{2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3} a}+\frac{2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{d\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3} a} \\
& +\frac{8 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3} a}+\frac{4 a}{d b^{5}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}-\frac{10 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right) a^{2}}{d b^{6}} \\
& -\frac{4 a}{d b^{5}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}+\frac{10 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) a^{2}}{d b^{6}}+\frac{18 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}} \\
& +\frac{12 a^{4}}{d b^{5}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}+\frac{5 a^{2}}{3 d b^{3}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}} \\
& +\frac{4 b \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3} a^{2}}+\frac{63 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}} \\
& +\frac{10 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d b^{2}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}-\frac{20 a^{3} \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{d b^{6} \sqrt{a^{2}+b^{2}}}-\frac{15 a \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{d b^{4} \sqrt{a^{2}+b^{2}}} \\
& +\frac{9 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}+\frac{12 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d b^{5}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}} \\
& -\frac{39 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d b^{3}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}-\frac{4 b \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3} a^{2}} \\
& -\frac{72 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}+\frac{38 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d b^{2}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{8 b^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3} a^{3}}-\frac{24 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d b^{5}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}} \\
& +\frac{100 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d^{3}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}+\frac{1}{2 d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2 d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)} \\
& -\frac{5 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2 d b^{4}}-\frac{1}{2 d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2 d b^{4}\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}+\frac{5 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{2 d b^{4}} \\
& +\frac{2}{3 d b\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{3}}
\end{aligned}
$$

Problem 50: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (x)}{\sec (x)+\tan (x)} \mathrm{d} x
$$

Optimal(type 3, 4 leaves, 3 steps):

$$
x+\cos (x)
$$

Result(type 3, 14 leaves):

$$
\frac{2}{\tan \left(\frac{x}{2}\right)^{2}+1}+x
$$

Problem 51: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (x)}{\sec (x)-\tan (x)} d x
$$

Optimal(type 3, 6 leaves, 3 steps):

$$
x-\cos (x)
$$

Result(type 3, 14 leaves):

$$
-\frac{2}{\tan \left(\frac{x}{2}\right)^{2}+1}+x
$$

[^2]$$
\int \frac{\tan (x)}{\cot (x)+\csc (x)} \mathrm{d} x
$$

Optimal(type 3, 7 leaves, 4 steps):

$$
-x+\operatorname{arctanh}(\sin (x))
$$

Result(type 3, 20 leaves):

$$
-\ln \left(\tan \left(\frac{x}{2}\right)-1\right)+\ln \left(\tan \left(\frac{x}{2}\right)+1\right)-x
$$

Problem 76: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos (x)^{3} \sin (x)^{2}}{a \cos (x)+b \sin (x)} d x
$$

Optimal(type 3, 161 leaves, 13 steps):

$$
\begin{aligned}
& \frac{a^{3} b^{2} x}{\left(a^{2}+b^{2}\right)^{3}}-\frac{a b^{2} x}{2\left(a^{2}+b^{2}\right)^{2}}+\frac{a x}{8\left(a^{2}+b^{2}\right)}-\frac{b \cos (x)^{4}}{4\left(a^{2}+b^{2}\right)}+\frac{a^{2} b^{3} \ln (a \cos (x)+b \sin (x))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{a b^{2} \cos (x) \sin (x)}{2\left(a^{2}+b^{2}\right)^{2}}+\frac{a \cos (x) \sin (x)}{8\left(a^{2}+b^{2}\right)}-\frac{a \cos (x)^{3} \sin (x)}{4\left(a^{2}+b^{2}\right)} \\
& \quad-\frac{a^{2} b \sin (x)^{2}}{2\left(a^{2}+b^{2}\right)^{2}}
\end{aligned}
$$

Result(type 3, 362 leaves):

$$
\begin{aligned}
& \frac{\tan (x)^{3} a^{5}}{8\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}-\frac{\tan (x)^{3} a^{3} b^{2}}{4\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}-\frac{3 \tan (x)^{3} a b^{4}}{8\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}+\frac{\tan (x)^{2} a^{4} b}{2\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}} \\
& \quad+\frac{\tan (x)^{2} a^{2} b^{3}}{2\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}-\frac{3 \tan (x) a^{3} b^{2}}{4\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}-\frac{5 \tan (x) a b^{4}}{8\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}-\frac{b^{5}}{8\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}} \\
& \quad+\frac{a^{4} b}{4\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}-\frac{a^{2} b^{3} \ln \left(\tan (x)^{2}+1\right)}{2\left(a^{2}+b^{2}\right)^{3}}+\frac{\arctan (\tan (x)) a^{5}}{8\left(a^{2}+b^{2}\right)^{3}\left(\tan (x)^{2}+1\right)^{2}}+\frac{3 \arctan (\tan (x)) a^{3} b^{2}}{4\left(a^{2}+b^{2}\right)^{3}} \\
& \\
& -\frac{3 \arctan (\tan (x)) a b^{4}}{8\left(a^{2}+b^{2}\right)^{3}}+\frac{b^{3} a^{2} \ln (\tan (x) b+a)}{\left(a^{2}+b^{2}\right)^{3}}
\end{aligned}
$$

Test results for the 107 problems in $44.7 .3(c+d x)^{\wedge} m$ trig^n trig^p.txt"
Problem 1: Unable to integrate problem.

$$
\int(d x+c)^{m} \cos (b x+a) \sin (b x+a) \mathrm{d} x
$$

Optimal(type 4, 131 leaves, 5 steps):

$$
-\frac{2^{-3-m} \mathrm{e}^{2 \mathrm{I}\left(a-\frac{b c}{d}\right)}(d x+c)^{m} \Gamma\left(1+m, \frac{-2 \mathrm{I} b(d x+c)}{d}\right)}{b\left(\frac{-\mathrm{I} b(d x+c)}{d}\right)^{m}}-\frac{2^{-3-m}(d x+c)^{m} \Gamma\left(1+m, \frac{2 \mathrm{I} b(d x+c)}{d}\right)}{b \mathrm{e}^{2 \mathrm{I}\left(a-\frac{b c}{d}\right)}\left(\frac{\mathrm{I} b(d x+c)}{d}\right)^{m}}
$$

Result(type 8, 22 leaves):

$$
\int(d x+c)^{m} \cos (b x+a) \sin (b x+a) \mathrm{d} x
$$

Problem 2: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a) \sin (b x+a) \mathrm{d} x
$$

Optimal(type 3, 108 leaves, 5 steps):
$\frac{3 d^{3} x}{8 b^{3}}-\frac{(d x+c)^{3}}{4 b}-\frac{3 d^{3} \cos (b x+a) \sin (b x+a)}{8 b^{4}}+\frac{3 d(d x+c)^{2} \cos (b x+a) \sin (b x+a)}{4 b^{2}}-\frac{3 d^{2}(d x+c) \sin (b x+a)^{2}}{4 b^{3}}+\frac{(d x+c)^{3} \sin (b x+a)^{2}}{2 b}$
Result(type 3, 465 leaves):

$$
\frac{1}{b}\left(\frac { 1 } { b ^ { 3 } } \left(d ^ { 3 } \left(-\frac{(b x+a)^{3} \cos (b x+a)^{2}}{2}+\frac{3(b x+a)^{2}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{2}+\frac{3(b x+a) \cos (b x+a)^{2}}{4}\right.\right.\right.
$$

$$
\left.\left.-\frac{3 \cos (b x+a) \sin (b x+a)}{8}-\frac{3 b x}{8}-\frac{3 a}{8}-\frac{(b x+a)^{3}}{2}\right)\right)
$$

$$
-\frac{3 a d^{3}\left(-\frac{(b x+a)^{2} \cos (b x+a)^{2}}{2}+(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{4}-\frac{\sin (b x+a)^{2}}{4}\right)}{b^{3}}
$$

$$
+\frac{3 c d^{2}\left(-\frac{(b x+a)^{2} \cos (b x+a)^{2}}{2}+(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{4}-\frac{\sin (b x+a)^{2}}{4}\right)}{b^{2}}
$$

$$
+\frac{3 a^{2} d^{3}\left(-\frac{(b x+a) \cos (b x+a)^{2}}{2}+\frac{\cos (b x+a) \sin (b x+a)}{4}+\frac{b x}{4}+\frac{a}{4}\right)}{b^{3}}
$$

$$
-\frac{6 a c d^{2}\left(-\frac{(b x+a) \cos (b x+a)^{2}}{2}+\frac{\cos (b x+a) \sin (b x+a)}{4}+\frac{b x}{4}+\frac{a}{4}\right)}{b^{2}}
$$

$$
+\frac{3 c^{2} d\left(-\frac{(b x+a) \cos (b x+a)^{2}}{2}+\frac{\cos (b x+a) \sin (b x+a)}{4}+\frac{b x}{4}+\frac{a}{4}\right)}{b}+\frac{a^{3} d^{3} \cos (b x+a)^{2}}{2 b^{3}}-\frac{3 a^{2} c d^{2} \cos (b x+a)^{2}}{2 b^{2}}
$$

$$
\left.+\frac{3 a c^{2} d \cos (b x+a)^{2}}{2 b}-\frac{c^{3} \cos (b x+a)^{2}}{2}\right)
$$

Problem 5: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a) \sin (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 3, 137 leaves, 7 steps):

$$
\begin{aligned}
& -\frac{14 d^{3} \cos (b x+a)}{9 b^{4}}+\frac{2 d(d x+c)^{2} \cos (b x+a)}{3 b^{2}}+\frac{2 d^{3} \cos (b x+a)^{3}}{27 b^{4}}-\frac{4 d^{2}(d x+c) \sin (b x+a)}{3 b^{3}}+\frac{d(d x+c)^{2} \cos (b x+a) \sin (b x+a)^{2}}{3 b^{2}} \\
& -\frac{2 d^{2}(d x+c) \sin (b x+a)^{3}}{9 b^{3}}+\frac{(d x+c)^{3} \sin (b x+a)^{3}}{3 b}
\end{aligned}
$$

Result(type 3, 446 leaves):
$\frac{1}{b}\left(\frac{1}{b^{3}}\left(d^{3}\left(\frac{(b x+a)^{3} \sin (b x+a)^{3}}{3}+\frac{(b x+a)^{2}\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{3}-\frac{4 \cos (b x+a)}{3}-\frac{4 \sin (b x+a)(b x+a)}{3}\right.\right.\right.$

$$
\left.\left.-\frac{2(b x+a) \sin (b x+a)^{3}}{9}-\frac{2\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{27}\right)\right)
$$

$$
-\frac{3 a d^{3}\left(\frac{(b x+a)^{2} \sin (b x+a)^{3}}{3}+\frac{2(b x+a)\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}-\frac{2 \sin (b x+a)^{3}}{27}-\frac{4 \sin (b x+a)}{9}\right)}{b^{3}}
$$

$$
+\frac{3 c d^{2}\left(\frac{(b x+a)^{2} \sin (b x+a)^{3}}{3}+\frac{2(b x+a)\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}-\frac{2 \sin (b x+a)^{3}}{27}-\frac{4 \sin (b x+a)}{9}\right)}{b^{2}}
$$

$$
+\frac{3 a^{2} d^{3}\left(\frac{(b x+a) \sin (b x+a)^{3}}{3}+\frac{\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}\right)}{b^{3}}-\frac{6 a c d^{2}\left(\frac{(b x+a) \sin (b x+a)^{3}}{3}+\frac{\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}\right)}{b^{2}}
$$

$$
+\frac{3 c^{2} d\left(\frac{(b x+a) \sin (b x+a)^{3}}{3}+\frac{\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}\right)}{b}-\frac{a^{3} d^{3} \sin (b x+a)^{3}}{3 b^{3}}+\frac{a^{2} c d^{2} \sin (b x+a)^{3}}{b^{2}}-\frac{a c^{2} d \sin (b x+a)^{3}}{b}
$$

$$
\left.+\frac{c^{3} \sin (b x+a)^{3}}{3}\right)
$$

[^3]$$
\int(d x+c)^{2} \cos (b x+a) \sin (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 3, 93 leaves, 4 steps):

$$
\frac{4 d(d x+c) \cos (b x+a)}{9 b^{2}}-\frac{4 d^{2} \sin (b x+a)}{9 b^{3}}+\frac{2 d(d x+c) \cos (b x+a) \sin (b x+a)^{2}}{9 b^{2}}-\frac{2 d^{2} \sin (b x+a)^{3}}{27 b^{3}}+\frac{(d x+c)^{2} \sin (b x+a)^{3}}{3 b}
$$

Result(type 3, 203 leaves):

$$
\begin{aligned}
& \frac{1}{b}\left(\frac{d^{2}\left(\frac{(b x+a)^{2} \sin (b x+a)^{3}}{3}+\frac{2(b x+a)\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}-\frac{2 \sin (b x+a)^{3}}{27}-\frac{4 \sin (b x+a)}{9}\right)}{b^{2}}+\frac{2 a d^{2}\left(\frac{(b x+a) \sin (b x+a)^{3}}{3}+\frac{\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}\right)}{b^{2}}+\frac{2 c d\left(\frac{(b x+a) \sin (b x+a)^{3}}{3}+\frac{\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{9}\right)}{b}\right. \\
& \left.\quad+\frac{a^{2} d^{2} \sin (b x+a)^{3}}{3 b^{2}}-\frac{2 a c d \sin (b x+a)^{3}}{3 b}+\frac{c^{2} \sin (b x+a)^{3}}{3}\right)
\end{aligned}
$$

Problem 10: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{4} \cos (b x+a) \sin (b x+a)^{3} \mathrm{~d} x
$$

Optimal(type 3, 236 leaves, 9 steps):

$$
\begin{aligned}
& \frac{45 c d^{3} x}{64 b^{3}}+\frac{45 d^{4} x^{2}}{128 b^{3}}-\frac{3(d x+c)^{4}}{32 b}-\frac{45 d^{3}(d x+c) \cos (b x+a) \sin (b x+a)}{64 b^{4}}+\frac{3 d(d x+c)^{3} \cos (b x+a) \sin (b x+a)}{8 b^{2}}+\frac{45 d^{4} \sin (b x+a)^{2}}{128 b^{5}} \\
& -\frac{9 d^{2}(d x+c)^{2} \sin (b x+a)^{2}}{16 b^{3}}-\frac{3 d^{3}(d x+c) \cos (b x+a) \sin (b x+a)^{3}}{32 b^{4}}+\frac{d(d x+c)^{3} \cos (b x+a) \sin (b x+a)^{3}}{4 b^{2}}+\frac{3 d^{4} \sin (b x+a)^{4}}{128 b^{5}} \\
& -\frac{3 d^{2}(d x+c)^{2} \sin (b x+a)^{4}}{16 b^{3}}+\frac{(d x+c)^{4} \sin (b x+a)^{4}}{4 b}
\end{aligned}
$$

Result(type 3, 1142 leaves):
$\frac{1}{b}\left(\frac{1}{b^{4}}\left(d^{4}\left(\frac{(b x+a)^{4} \sin (b x+a)^{4}}{4}-(b x+a)^{3}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{3(b x+a)^{2} \sin (b x+a)^{4}}{16}\right.\right.\right.$

$$
+\frac{3(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{8}+\frac{27(b x+a)^{2}}{128}+\frac{3 \sin (b x+a)^{4}}{128}+\frac{45 \sin (b x+a)^{2}}{128}
$$

$$
\begin{aligned}
& \left.\left.+\frac{9(b x+a)^{2} \cos (b x+a)^{2}}{16}-\frac{9(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{8}+\frac{9(b x+a)^{4}}{32}\right)\right) \\
& -\frac{1}{b^{4}}\left(4 a d ^ { 4 } \left(\frac{(b x+a)^{3} \sin (b x+a)^{4}}{4}-\frac{3(b x+a)^{2}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{4}\right.\right. \\
& -\frac{3(b x+a) \sin (b x+a)^{4}}{32}-\frac{3\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{2}-\frac{27 b x}{256}-\frac{27 a}{256}+\frac{9(b x+a) \cos (b x+a)^{2}}{32} \\
& \left.\left.-\frac{9 \cos (b x+a) \sin (b x+a)}{64}+\frac{3(b x+a)^{3}}{16}\right)\right)+\frac{1}{b^{3}}\left(4 c d ^ { 3 } \left(\frac{(b x+a)^{3} \sin (b x+a)^{4}}{4}\right.\right. \\
& -\frac{3(b x+a)^{2}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{4}+\frac{3 a}{8}\right)}{4}-\frac{3(b x+a) \sin (b x+a)^{4}}{32}
\end{aligned}
$$

$$
\left.\left.-\frac{3\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{128}-\frac{27 b x}{256}-\frac{27 a}{256}+\frac{9(b x+a) \cos (b x+a)^{2}}{32}-\frac{9 \cos (b x+a) \sin (b x+a)}{64}+\frac{3(b x+a)^{3}}{16}\right)\right)
$$

$$
+\frac{1}{b^{4}}\left(6 a ^ { 2 } d ^ { 4 } \left(\frac{(b x+a)^{2} \sin (b x+a)^{4}}{4}-\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{2}+\frac{3(b x+a)^{2}}{32}\right.\right.
$$

$$
\left.\left.-\frac{\sin (b x+a)^{4}}{32}-\frac{3 \sin (b x+a)^{2}}{32}\right)\right)-\frac{1}{b^{3}}\left(1 2 a c d ^ { 3 } \left(\frac{(b x+a)^{2} \sin (b x+a)^{4}}{4}\right.\right.
$$

$$
\begin{aligned}
& \left.\left.-\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{2}+\frac{3(b x+a)^{2}}{32}-\frac{\sin (b x+a)^{4}}{32}-\frac{3 \sin (b x+a)^{2}}{32}\right)\right) \\
& +\frac{1}{b^{2}}\left(6 c ^ { 2 } d ^ { 2 } \left(\frac{(b x+a)^{2} \sin (b x+a)^{4}}{4}-\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{2}+\frac{3(b x+a)^{2}}{32}\right.\right. \\
& \left.\left.-\frac{\sin (b x+a)^{4}}{32}-\frac{3 \sin (b x+a)^{2}}{32}\right)\right)-\frac{4 a^{3} d^{4}\left(\frac{(b x+a) \sin (b x+a)^{4}}{4}+\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{3 b x}{32}-\frac{3 a}{32}\right)}{b^{4}} \\
& +\frac{12 a^{2} c d^{3}\left(\frac{(b x+a) \sin (b x+a)^{4}}{4}+\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{3 b x}{32}-\frac{3 a}{32}\right)}{b^{3}} \\
& -\frac{12 a c^{2} d^{2}\left(\frac{(b x+a) \sin (b x+a)^{4}}{4}+\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{3 b x}{32}-\frac{3 a}{32}\right)}{b^{2}} \\
& +\frac{4 c^{3} d\left(\frac{(b x+a) \sin (b x+a)^{4}}{4}+\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{3 b x}{32}-\frac{3 a}{32}\right)}{b}+\frac{a^{4} d^{4} \sin (b x+a)^{4}}{4 b^{4}}-\frac{a^{3} c d^{3} \sin (b x+a)^{4}}{b^{3}} \\
& \left.+\frac{3 a^{2} c^{2} d^{2} \sin (b x+a)^{4}}{2 b^{2}}-\frac{a c^{3} d \sin (b x+a)^{4}}{b}+\frac{c^{4} \sin (b x+a)^{4}}{4}\right)
\end{aligned}
$$

Problem 12: Result more than twice size of optimal antiderivative.

$$
\int(d x+c) \cos (b x+a) \csc (b x+a) \mathrm{d} x
$$

Optimal(type 4, 55 leaves, 4 steps):

$$
-\frac{\mathrm{I}(d x+c)^{2}}{2 d}+\frac{(d x+c) \ln \left(1-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}-\frac{\mathrm{I} d \operatorname{polylog}\left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}
$$

Result(type 4, 214 leaves):

$$
-\frac{\mathrm{I} d x^{2}}{2}+\mathrm{I} c x-\frac{2 c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}-\frac{2 \mathrm{I} d a x}{b}-\frac{\mathrm{I} d a^{2}}{b^{2}}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}
$$

$$
-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{2 a d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{a d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}
$$

Problem 13: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{4} \cos (b x+a) \csc (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 4, 190 leaves, 10 steps):

$$
\begin{array}{r}
-\frac{8 d(d x+c)^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{(d x+c)^{4} \csc (b x+a)}{b}+\frac{12 \mathrm{I} d^{2}(d x+c)^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{12 \mathrm{I} d^{2}(d x+c)^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
-\frac{24 d^{3}(d x+c) \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{24 d^{3}(d x+c) \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}+\frac{24 \mathrm{I} d^{4} \operatorname{polylog}\left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}
\end{array}
$$

Result(type 4, 715 leaves):

$$
\begin{aligned}
& -\frac{2 \mathrm{I}\left(d^{4} x^{4}+4 c d^{3} x^{3}+6 c^{2} d^{2} x^{2}+4 c^{3} d x+c^{4}\right) \mathrm{e}^{\mathrm{I}(b x+a)}}{b\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)}-\frac{12 \mathrm{I} d^{4} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{3}}+\frac{12 \mathrm{I} d^{4} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{3}} \\
& -\frac{12 \mathrm{I} d^{2} c^{2} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{12 \mathrm{I} d^{2} c^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}-\frac{24 \mathrm{I} d^{3} c \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}} \\
& +\frac{24 \mathrm{I} d^{3} c \text { polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{24 d^{4} \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{4}}-\frac{8 d c^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{8 d^{4} a^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}} \\
& +\frac{24 d^{3} c \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{24 d^{3} c \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{24 d^{4} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{4}}-\frac{12 d^{2} c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b^{2}} \\
& -\frac{12 d^{2} c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a}{b^{3}}+\frac{12 d^{2} c^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{12 d^{2} c^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}+\frac{12 d^{3} c a^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b^{4}}-\frac{12 d^{3} c a^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& +\frac{12 d^{3} c \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{12 d^{3} c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{2}}{b^{2}}-\frac{4 d^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{3}}{b^{2}}-\frac{4 d^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a^{3}}{b^{5}}+\frac{4 d^{4} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{b^{2}} \\
& +\frac{4 d^{4} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{3}}{b^{5}}-\frac{24 d^{3} a^{2} c \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{24 d^{2} a c^{2} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}
\end{aligned}
$$

Problem 14: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \cos (b x+a) \csc (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 4, 82 leaves, 6 steps):

$$
-\frac{4 d(d x+c) \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{(d x+c)^{2} \csc (b x+a)}{b}+\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}
$$

Result(type 4, 211 leaves):
$-\frac{2 \mathrm{I}\left(x^{2} d^{2}+2 c d x+c^{2}\right) \mathrm{e}^{\mathrm{I}(b x+a)}}{b\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)}-\frac{4 d c \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{2 d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{2 d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}-\frac{2 \mathrm{I} d^{2} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}$

$$
-\frac{2 d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b^{2}}-\frac{2 d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a}{b^{3}}+\frac{2 \mathrm{I} d^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{4 d^{2} a \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}
$$

Problem 16: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a) \csc (b x+a)^{3} \mathrm{~d} x
$$

Optimal(type 4, 101 leaves, 6 steps):

$$
-\frac{3 \mathrm{I} d(d x+c)^{2}}{2 b^{2}}-\frac{3 d(d x+c)^{2} \cot (b x+a)}{2 b^{2}}-\frac{(d x+c)^{3} \csc (b x+a)^{2}}{2 b}+\frac{3 d^{2}(d x+c) \ln \left(1-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{4}}
$$

Result(type 4, 408 leaves):

$$
\begin{aligned}
& \frac{1}{b^{2}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)^{2}}\left(2 b d^{3} x^{3} \mathrm{e}^{2 \mathrm{I}(b x+a)}-3 \mathrm{I} d^{3} x^{2} \mathrm{e}^{2 \mathrm{I}(b x+a)}+6 b c d^{2} x^{2} \mathrm{e}^{2 \mathrm{I}(b x+a)}-6 \mathrm{I} c d^{2} x \mathrm{e}^{2 \mathrm{I}(b x+a)}+6 b c^{2} d x \mathrm{e}^{2 \mathrm{I}(b x+a)}-3 \mathrm{I} c^{2} d \mathrm{e}^{2 \mathrm{I}(b x+a)}+3 \mathrm{I} d^{3} x^{2}\right. \\
& \left.\quad+2 b c^{3} \mathrm{e}^{\mathrm{I}(b x+a)}+6 \mathrm{I} c d^{2} x+3 \mathrm{I} c^{2} d\right)-\frac{6 d^{2} c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 d^{2} c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}+\frac{3 d^{2} c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b^{3}}-\frac{3 \mathrm{I} d^{3} x^{2}}{b^{2}}-\frac{6 \mathrm{I} d^{3} a x}{b^{3}}-\frac{3 \mathrm{I} d^{3} a^{2}}{b^{4}} \\
& +\frac{3 d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{3 d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{4}}-\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{3 d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b^{3}}-\frac{3 \mathrm{I} d^{3} p o l y \log \left(2,--\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& \quad+\frac{6 d^{3} a \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 d^{3} a \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{4}}
\end{aligned}
$$

Problem 23: Unable to integrate problem.

$$
\int(d x+c)^{m} \cos (b x+a)^{2} \sin (b x+a) \mathrm{d} x
$$

Optimal(type 4, 247 leaves, 8 steps):

$$
\begin{aligned}
& -\frac{\mathrm{e}^{\mathrm{I}\left(a-\frac{b c}{d}\right)}(d x+c)^{m} \Gamma\left(1+m, \frac{-\mathrm{I} b(d x+c)}{d}\right)}{8 b\left(\frac{-\mathrm{I} b(d x+c)}{d}\right)^{m}}-\frac{(d x+c)^{m} \Gamma\left(1+m, \frac{\mathrm{I} b(d x+c)}{d}\right)}{8 b \mathrm{e}^{\mathrm{I}\left(a-\frac{b c}{d}\right)}\left(\frac{\mathrm{I} b(d x+c)}{d}\right)^{m}}-\frac{3^{-1-m} \mathrm{e}^{3 \mathrm{I}\left(a-\frac{b c}{d}\right)}(d x+c)^{m} \Gamma\left(1+m, \frac{-3 \mathrm{I} b(d x+c)}{d}\right)}{8 b\left(\frac{-\mathrm{I} b(d x+c)}{d}\right)^{m}} \\
& \quad-\frac{3^{-1-m}(d x+c)^{m} \Gamma\left(1+m, \frac{3 \mathrm{I} b(d x+c)}{d}\right)}{8 b \mathrm{e}^{3 \mathrm{I}\left(a-\frac{b c}{d}\right)}\left(\frac{\mathrm{I} b(d x+c)}{d}\right)^{m}}
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int(d x+c)^{m} \cos (b x+a)^{2} \sin (b x+a) \mathrm{d} x
$$

[^4]$$
\int(d x+c)^{4} \cos (b x+a)^{2} \sin (b x+a)^{2} \mathrm{~d} x
$$
\[

$$
\begin{aligned}
& \text { Optimal (type 3, } 119 \text { leaves, } 7 \text { steps): } \\
& \frac{(d x+c)^{5}}{40 d}+\frac{3 d^{3}(d x+c) \cos (4 b x+4 a)}{256 b^{4}}-\frac{d( }{32 b} \\
& \quad-\frac{(d x+c)^{4} \sin (4 b x+4 a)}{32}
\end{aligned}
$$
\]

$$
\frac{(d x+c)^{5}}{40 d}+\frac{3 d^{3}(d x+c) \cos (4 b x+4 a)}{256 b^{4}}-\frac{d(d x+c)^{3} \cos (4 b x+4 a)}{32 b^{2}}-\frac{3 d^{4} \sin (4 b x+4 a)}{1024 b^{5}}+\frac{3 d^{2}(d x+c)^{2} \sin (4 b x+4 a)}{128 b^{3}}
$$

Result(type 3, 1914 leaves):

$$
\frac{1}{b}\left(\frac { 1 } { b ^ { 4 } } \left(d ^ { 4 } \left((b x+a)^{4}\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{3} \cos (b x+a)^{2}}{4}+\frac{3(b x+a)^{2}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{4}\right.\right.\right.
$$

$$
+\frac{3(b x+a) \cos (b x+a)^{2}}{32}-\frac{3 \cos (b x+a) \sin (b x+a)}{64}-\frac{21 b x}{256}-\frac{21 a}{256}-\frac{7(b x+a)^{3}}{16}-\frac{(b x+a)^{5}}{10}-(b x+a)^{4}(
$$

$$
\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{(b x+a)^{3} \sin (b x+a)^{4}}{4}
$$

$$
+\frac{3(b x+a)^{2}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{4}+\frac{3(b x+a) \sin (b x+a)^{4}}{32}
$$

$$
\left.\left.+\frac{3\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{128}\right)\right)-\frac{1}{b^{4}}\left(4 a d ^ { 4 } \left((b x+a)^{3}\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right.\right.
$$

$$
-\frac{3(b x+a)^{2} \cos (b x+a)^{2}}{16}+\frac{3(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{8}-\frac{21(b x+a)^{2}}{128}-\frac{3 \sin (b x+a)^{2}}{128}-\frac{3(b x+a)^{4}}{32}
$$

$-(b x+a)^{3}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{3(b x+a)^{2} \sin (b x+a)^{4}}{16}$
$\left.\left.+\frac{3(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{8}+\frac{3 \sin (b x+a)^{4}}{128}\right)\right)+\frac{1}{b^{3}}\left(4 c d^{3}\left((b x+a)^{3}(\right.\right.$
$\left.-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{3(b x+a)^{2} \cos (b x+a)^{2}}{16}+\frac{3(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{8}-\frac{21(b x+a)^{2}}{128}$
$-\frac{3 \sin (b x+a)^{2}}{128}-\frac{3(b x+a)^{4}}{32}-(b x+a)^{3}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{3(b x+a)^{2} \sin (b x+a)^{4}}{16}$
$\left.\left.+\frac{3(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{8}+\frac{3 \sin (b x+a)^{4}}{128}\right)\right)+\frac{1}{b^{4}}\left(6 a^{2} d^{4}\left((b x+a)^{2}(\right.\right.$
$\left.-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a) \cos (b x+a)^{2}}{8}+\frac{\cos (b x+a) \sin (b x+a)}{16}+\frac{7 b x}{64}+\frac{7 a}{64}-\frac{(b x+a)^{3}}{12}-(b x+a)^{2}($
$\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{(b x+a) \sin (b x+a)^{4}}{8}$
$\left.\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{32}\right)\right)-\frac{1}{b^{3}}\left(12 a c d^{3}\left((b x+a)^{2}\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right.\right.$
$-\frac{(b x+a) \cos (b x+a)^{2}}{8}+\frac{\cos (b x+a) \sin (b x+a)}{16}+\frac{7 b x}{64}+\frac{7 a}{64}-\frac{(b x+a)^{3}}{12}-(b x+a)^{2}$
$\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{(b x+a) \sin (b x+a)^{4}}{8}$
$\left.\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{32}\right)\right)+\frac{1}{b^{2}}\left(6 c^{2} d^{2}\left((b x+a)^{2}\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right.\right.$
$-\frac{(b x+a) \cos (b x+a)^{2}}{8}+\frac{\cos (b x+a) \sin (b x+a)}{16}+\frac{7 b x}{64}+\frac{7 a}{64}-\frac{(b x+a)^{3}}{12}-(b x+a)^{2}$
$\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{(b x+a) \sin (b x+a)^{4}}{8}$
$\left.\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{32}\right)\right)-\frac{1}{b^{4}}\left(4 a^{3} d^{4}\left((b x+a)\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}\right.\right.$
$\left.\left.+\frac{\sin (b x+a)^{2}}{16}-(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)+\frac{1}{b^{3}}\left(12 a^{2} c d^{3}((b x\right.$
$+a)\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}+\frac{\sin (b x+a)^{2}}{16}-(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}\right.$
$\left.\left.\left.+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)-\frac{1}{b^{2}}\left(12 a c^{2} d^{2}\left((b x+a)\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}+\frac{\sin (b x+a)^{2}}{16}\right.\right.$
$\left.\left.-(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)+\frac{1}{b}\left(4 c^{3} d((b x+a)(\right.$
$\left.-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}+\frac{\sin (b x+a)^{2}}{16}-(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}\right.$

$$
\begin{aligned}
& \left.\left.\left.+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)+\frac{a^{4} d^{4}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b^{4}} \\
& -\frac{4 a^{3} c d^{3}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b^{3}} \\
& +\frac{6 a^{2} c^{2} d^{2}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b^{2}} \\
& -\frac{4 a c^{3} d\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b}+c^{4}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}\right. \\
& \left.+\frac{b x}{8}+\frac{a}{8}\right)
\end{aligned}
$$

Problem 25: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a)^{2} \sin (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 3, 95 leaves, 6 steps):

$$
\frac{(d x+c)^{4}}{32 d}+\frac{3 d^{3} \cos (4 b x+4 a)}{1024 b^{4}}-\frac{3 d(d x+c)^{2} \cos (4 b x+4 a)}{128 b^{2}}+\frac{3 d^{2}(d x+c) \sin (4 b x+4 a)}{256 b^{3}}-\frac{(d x+c)^{3} \sin (4 b x+4 a)}{32 b}
$$

Result(type 3, 1073 leaves):
$\frac{1}{b}\left(\frac{1}{b^{3}}\left(d^{3}\left((b x+a)^{3}\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{3(b x+a)^{2} \cos (b x+a)^{2}}{16}\right.\right.\right.$
$+\frac{3(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{8}-\frac{21(b x+a)^{2}}{128}-\frac{3 \sin (b x+a)^{2}}{128}-\frac{3(b x+a)^{4}}{32}-(b x+a)^{3}($

$$
\begin{aligned}
& \left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{3(b x+a)^{2} \sin (b x+a)^{4}}{16} \\
& \left.\left.+\frac{3(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{8}+\frac{3 \sin (b x+a)^{4}}{128}\right)\right)-\frac{1}{b^{3}}\left(3 a d ^ { 3 } \left((b x+a)^{2}( \right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a) \cos (b x+a)^{2}}{8}+\frac{\cos (b x+a) \sin (b x+a)}{16}+\frac{7 b x}{64}+\frac{7 a}{64}-\frac{(b x+a)^{3}}{12}-(b x+a)^{2}( \\
& \left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{(b x+a) \sin (b x+a)^{4}}{8} \\
& \left.\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{32}\right)\right)+\frac{1}{b^{2}}\left(3 c d ^ { 2 } \left((b x+a)^{2}\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right.\right. \\
& -\frac{(b x+a) \cos (b x+a)^{2}}{8}+\frac{\cos (b x+a) \sin (b x+a)}{16}+\frac{7 b x}{64}+\frac{7 a}{64}-\frac{(b x+a)^{3}}{12}-(b x+a)^{2} \\
& \left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{(b x+a) \sin (b x+a)^{4}}{8} \\
& \left.\left.-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{32}\right)\right)+\frac{1}{b^{3}}\left(3 a ^ { 2 } d ^ { 3 } \left((b x+a)\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}\right.\right. \\
& \left.\left.+\frac{\sin (b x+a)^{2}}{16}-(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)-\frac{1}{b^{2}}\left(6 a c d^{2}((b x\right. \\
& +a)\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}+\frac{\sin (b x+a)^{2}}{16}-(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}\right. \\
& \left.\left.\left.+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)+\frac{1}{b}\left(3 c ^ { 2 } d \left((b x+a)\left(-\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{16}+\frac{\sin (b x+a)^{2}}{16}-(b x\right.\right. \\
& \left.\left.+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)-\frac{\sin (b x+a)^{4}}{16}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{a^{3} d^{3}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b^{3}} \\
& +\frac{3 a^{2} c d^{2}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b^{2}} \\
& -\frac{3 a c^{2} d\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}+\frac{b x}{8}+\frac{a}{8}\right)}{b}+c^{3}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)}{4}+\frac{\cos (b x+a) \sin (b x+a)}{8}\right. \\
& \left.\left.+\frac{b x}{8}+\frac{a}{8}\right)\right)
\end{aligned}
$$

Problem 26: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \cos (b x+a)^{2} \sin (b x+a)^{3} \mathrm{~d} x
$$

Optimal(type 3, 166 leaves, 11 steps):

$$
\begin{aligned}
& \frac{d^{2} \cos (b x+a)}{4 b^{3}}-\frac{(d x+c)^{2} \cos (b x+a)}{8 b}+\frac{d^{2} \cos (3 b x+3 a)}{216 b^{3}}-\frac{(d x+c)^{2} \cos (3 b x+3 a)}{48 b}-\frac{d^{2} \cos (5 b x+5 a)}{1000 b^{3}}+\frac{(d x+c)^{2} \cos (5 b x+5 a)}{80 b} \\
& \quad+\frac{d(d x+c) \sin (b x+a)}{4 b^{2}}+\frac{d(d x+c) \sin (3 b x+3 a)}{72 b^{2}}-\frac{d(d x+c) \sin (5 b x+5 a)}{200 b^{2}}
\end{aligned}
$$

Result(type 3, 465 leaves):

$$
\begin{aligned}
& \frac{1}{b}\left(\frac { 1 } { b ^ { 2 } } \left(d ^ { 2 } \left(-\frac{(b x+a)^{2}\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{3}+\frac{4 \cos (b x+a)}{15}+\frac{4 \sin (b x+a)(b x+a)}{15}+\frac{2(b x+a) \sin (b x+a)^{3}}{45}\right.\right.\right. \\
& \quad+\frac{2\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{135}+\frac{(b x+a)^{2}\left(\frac{8}{3}+\sin (b x+a)^{4}+\frac{4 \sin (b x+a)^{2}}{3}\right) \cos (b x+a)}{5}-\frac{2(b x+a) \sin (b x+a)^{5}}{25} \\
& \left.\left.\quad-\frac{2\left(\frac{8}{3}+\sin (b x+a)^{4}+\frac{4 \sin (b x+a)^{2}}{3}\right) \cos (b x+a)}{3}\right)\right)-\frac{1}{b^{2}\left(2 a d ^ { 2 } \left(-\frac{(b x+a)\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{3}+\frac{\sin (b x+a)^{3}}{45}\right.\right.} \\
& \left.\left.\quad+\frac{2 \sin (b x+a)}{15}+\frac{(b x+a)\left(\frac{8}{3}+\sin (b x+a)^{4}+\frac{4 \sin (b x+a)^{2}}{3}\right) \cos (b x+a)}{5}-\frac{\sin (b x+a)^{5}}{25}\right)\right)+\frac{1}{b}(2 c d(
\end{aligned}
$$

$-\frac{(b x+a)\left(2+\sin (b x+a)^{2}\right) \cos (b x+a)}{3}+\frac{\sin (b x+a)^{3}}{45}+\frac{2 \sin (b x+a)}{15}+\frac{(b x+a)\left(\frac{8}{3}+\sin (b x+a)^{4}+\frac{4 \sin (b x+a)^{2}}{3}\right) \cos (b x+a)}{5}$

$$
\begin{aligned}
& \left.\left.-\frac{\sin (b x+a)^{5}}{25}\right)\right)+\frac{a^{2} d^{2}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)^{2}}{5}-\frac{2 \cos (b x+a)^{3}}{15}\right)}{b^{2}}-\frac{2 a c d\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)^{2}}{5}-\frac{2 \cos (b x+a)^{3}}{15}\right)}{b} \\
& \left.+c^{2}\left(-\frac{\cos (b x+a)^{3} \sin (b x+a)^{2}}{5}-\frac{2 \cos (b x+a)^{3}}{15}\right)\right)
\end{aligned}
$$

Problem 28: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \cos (b x+a) \cot (b x+a) \mathrm{d} x
$$

Optimal(type 4, 159 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{2(d x+c)^{2} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}-\frac{2 d^{2} \cos (b x+a)}{b^{3}}+\frac{(d x+c)^{2} \cos (b x+a)}{b}+\frac{2 \mathrm{I} d(d x+c) \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}} \\
& -\frac{2 \mathrm{I} d(d x+c) \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 d(d x+c) \sin (b x+a)}{b^{2}}
\end{aligned}
$$

Result(type 4, 478 leaves):
$\frac{\left(d^{2} x^{2} b^{2}+2 b^{2} c d x+b^{2} c^{2}+2 \mathrm{I} b d^{2} x-2 d^{2}+2 \mathrm{I} b c d\right) \mathrm{e}^{\mathrm{I}(b x+a)}}{2 b^{3}}+\frac{\left(d^{2} x^{2} b^{2}+2 b^{2} c d x+b^{2} c^{2}-2 \mathrm{I} b d^{2} x-2 d^{2}-2 \mathrm{I} b c d\right) \mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b^{3}}$

$$
\begin{aligned}
& -\frac{2 a^{2} d^{2} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 \mathrm{I} c d \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{2 d^{2} \operatorname{poly\operatorname {log}(3,\mathrm {e}^{\mathrm {I}(bx+a)})}}{b^{3}}-\frac{2 d^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b} \\
& -\frac{2 c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a}{b^{2}}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}+\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b}-\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2}}{b^{3}} \\
& +\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{2}}{b}+\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a^{2}}{b^{3}}-\frac{2 \mathrm{I} d^{2} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{2 c^{2} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b} \\
& +\frac{4 a c d \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}
\end{aligned}
$$

Problem 29: Result more than twice size of optimal antiderivative.

$$
\int(d x+c) \cos (b x+a) \cot (b x+a) \mathrm{d} x
$$

Optimal(type 4, 86 leaves, 8 steps):

$$
-\frac{2(d x+c) \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{(d x+c) \cos (b x+a)}{b}+\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d \sin (b x+a)}{b^{2}}
$$

Result(type 4, 202 leaves):
$\frac{(b d x+c b+\mathrm{I} d) \mathrm{e}^{\mathrm{I}(b x+a)}}{2 b^{2}}+\frac{(b d x+c b-\mathrm{I} d) \mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b^{2}}-\frac{2 c \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}$

$$
-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}-\frac{d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a}{b^{2}}+\frac{\mathrm{I} d \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{2 a d \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}
$$

Problem 31: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cot (b x+a)^{2} \csc (b x+a) \mathrm{d} x
$$

Optimal(type 4, 274 leaves, 25 steps):

$$
\begin{aligned}
&-\frac{6 d^{2}(d x+c) \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{(d x+c)^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}-\frac{3 d(d x+c)^{2} \csc (b x+a)}{2 b^{2}}-\frac{(d x+c)^{3} \cot (b x+a) \csc (b x+a)}{2 b} \\
&+\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 \mathrm{I} d(d x+c)^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{2 b^{2}}-\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{3 \mathrm{I} d(d x+c)^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{2 b^{2}} \\
&+\frac{3 d^{2}(d x+c) \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{3 d^{2}(d x+c) \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}
\end{aligned}
$$

Result(type 4, 1055 leaves):
$\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c^{2} d x}{2 b}-\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a c^{2} d}{2 b^{2}}-\frac{3 c^{2} d a \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{3}}{2 b}$
$+\frac{d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a^{3}}{2 b^{4}}-\frac{d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{2 b}-\frac{d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{3}}{2 b^{4}}+\frac{3 c d^{2} a^{2} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) c^{2} d x}{2 b}$
$+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a c^{2} d}{2 b^{2}}-\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c d^{2} x^{2}}{2 b}+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) c d^{2} x^{2}}{2 b}+\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2} c d^{2}}{2 b^{3}}-\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a^{2} c d^{2}}{2 b^{3}}$
$+\frac{3 \mathrm{I} c^{2} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{2 b^{2}}-\frac{3 \mathrm{I} c^{2} d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{2 b^{2}}+\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{2 b^{2}}-\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{2 b^{2}}$
$+\frac{3 d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{3 d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{4}}-\frac{3 d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b^{3}}-\frac{3 d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) a}{b^{4}}+\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}$
$+\frac{3 \mathrm{I} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) c d^{2} x}{b^{2}}-\frac{3 \mathrm{I} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c d^{2} x}{b^{2}}-\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}$
$+\frac{3 d^{3} \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}-\frac{3 d^{3} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}-\frac{6 c d^{2} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{3 c d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 c d^{2} p o l y \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}$
$+\frac{6 d^{3} a \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{d^{3} a^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{c^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{1}{b^{2}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)^{2}}\left(x^{3} d^{3} b \mathrm{e}^{3 \mathrm{I}(b x+a)}+3 c d^{2} x^{2} b \mathrm{e}^{3 \mathrm{I}(b x+a)}\right.$
$+3 c^{2} d x b \mathrm{e}^{3 \mathrm{I}(b x+a)}+x^{3} d^{3} b \mathrm{e}^{\mathrm{I}(b x+a)}+c^{3} b \mathrm{e}^{3 \mathrm{I}(b x+a)}+3 c d^{2} x^{2} b \mathrm{e}^{\mathrm{I}(b x+a)}-3 \mathrm{I} d^{3} x^{2} \mathrm{e}^{3 \mathrm{I}(b x+a)}+3 c^{2} d x b \mathrm{e}^{\mathrm{I}(b x+a)}-6 \mathrm{I} c d^{2} x \mathrm{e}^{3 \mathrm{I}(b x+a)}+c^{3} b \mathrm{e}^{\mathrm{I}(b x+a)}$
$\left.-3 \mathrm{I} c^{2} d \mathrm{e}^{3 \mathrm{I}(b x+a)}+3 \mathrm{I} d^{3} x^{2} \mathrm{e}^{\mathrm{I}(b x+a)}+6 \mathrm{I} c d^{2} x \mathrm{e}^{\mathrm{I}(b x+a)}+3 \mathrm{I} c^{2} d \mathrm{e}^{\mathrm{I}(b x+a)}\right)$

## Problem 42: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a)^{3} \sin (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 3, 235 leaves, 14 steps):

$$
\begin{array}{r}
-\frac{3 d^{3} \cos (b x+a)}{4 b^{4}}+\frac{3 d(d x+c)^{2} \cos (b x+a)}{8 b^{2}}+\frac{d^{3} \cos (3 b x+3 a)}{216 b^{4}}-\frac{d(d x+c)^{2} \cos (3 b x+3 a)}{48 b^{2}}+\frac{3 d^{3} \cos (5 b x+5 a)}{5000 b^{4}} \\
-\frac{3 d(d x+c)^{2} \cos (5 b x+5 a)}{400 b^{2}}-\frac{3 d^{2}(d x+c) \sin (b x+a)}{4 b^{3}}+\frac{(d x+c)^{3} \sin (b x+a)}{8 b}+\frac{d^{2}(d x+c) \sin (3 b x+3 a)}{72 b^{3}} \\
-\frac{(d x+c)^{3} \sin (3 b x+3 a)}{48 b}+\frac{3 d^{2}(d x+c) \sin (5 b x+5 a)}{1000 b^{3}}-\frac{(d x+c)^{3} \sin (5 b x+5 a)}{80 b}
\end{array}
$$

Result (type 3, 1015 leaves):

$$
\begin{aligned}
& \frac{1}{b}\left(\frac { 1 } { b ^ { 3 } } \left(d ^ { 3 } \left(\frac{(b x+a)^{3}\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}+\frac{2(b x+a)^{2} \cos (b x+a)}{5}-\frac{856 \cos (b x+a)}{1125}-\frac{4 \sin (b x+a)(b x+a)}{5}\right.\right.\right. \\
& \quad+\frac{(b x+a)^{2} \cos (b x+a)^{3}}{15}-\frac{2(b x+a)\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{45}+\frac{22 \cos (b x+a)^{3}}{3375}
\end{aligned}
$$

$$
-\frac{(b x+a)^{3}\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{3(b x+a)^{2} \cos (b x+a)^{5}}{25}
$$

$$
\left.\left.+\frac{6(b x+a)\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{125}+\frac{6 \cos (b x+a)^{5}}{625}\right)\right)
$$

$$
-\frac{1}{b^{3}}\left(3 a d ^ { 3 } \left(\frac{(b x+a)^{2}\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}-\frac{4 \sin (b x+a)}{15}+\frac{4(b x+a) \cos (b x+a)}{15}+\frac{2(b x+a) \cos (b x+a)^{3}}{45}\right.\right.
$$

$$
-\frac{2\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{135}-\frac{(b x+a)^{2}\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{2(b x+a) \cos (b x+a)^{5}}{25}
$$

$\left.\left.+\frac{2\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{125}\right)\right)+\frac{1}{b^{2}}\left(3 c d^{2}\left(\frac{(b x+a)^{2}\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}-\frac{4 \sin (b x+a)}{15}\right.\right.$
$+\frac{4(b x+a) \cos (b x+a)}{15}+\frac{2(b x+a) \cos (b x+a)^{3}}{45}-\frac{2\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{135}$
$-\frac{(b x+a)^{2}\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{2(b x+a) \cos (b x+a)^{5}}{25}$
$\left.\left.+\frac{2\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{125}\right)\right)+\frac{1}{b^{3}}\left(3 a^{2} d^{3}\left(\frac{(b x+a)\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}+\frac{\cos (b x+a)^{3}}{45}\right.\right.$
$\left.\left.+\frac{2 \cos (b x+a)}{15}-\frac{(b x+a)\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{\cos (b x+a)^{5}}{25}\right)\right)$
$-\frac{1}{b^{2}}\left(6 a c d^{2}\left(\frac{(b x+a)\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}+\frac{\cos (b x+a)^{3}}{45}+\frac{2 \cos (b x+a)}{15}\right.\right.$
$\left.\left.-\frac{(b x+a)\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{\cos (b x+a)^{5}}{25}\right)\right)$
$+\frac{1}{b}\left(3 c^{2} d\left(\frac{(b x+a)\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}+\frac{\cos (b x+a)^{3}}{45}+\frac{2 \cos (b x+a)}{15}\right.\right.$
$\left.\left.-\frac{(b x+a)\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{\cos (b x+a)^{5}}{25}\right)\right)$
$-\frac{a^{3} d^{3}\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)}{b^{3}}$

$$
\begin{aligned}
& +\frac{3 a^{2} c d^{2}\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)}{b^{2}} \\
& -\frac{3 a c^{2} d\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)}{b}+c^{3}\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}\right. \\
& \left.\left.+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)\right)
\end{aligned}
$$

Problem 43: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \cos (b x+a)^{3} \sin (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 3, 166 leaves, 11 steps):


Result(type 3, 483 leaves):
$\frac{1}{b}\left(\frac{1}{b^{2}}\left(d^{2}\left(\frac{(b x+a)^{2}\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}-\frac{4 \sin (b x+a)}{15}+\frac{4(b x+a) \cos (b x+a)}{15}+\frac{2(b x+a) \cos (b x+a)^{3}}{45}\right.\right.\right.$
$-\frac{2\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{135}-\frac{(b x+a)^{2}\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{2(b x+a) \cos (b x+a)^{5}}{25}$
$\left.\left.+\frac{2\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{125}\right)\right)-\frac{1}{b^{2}}\left(2 a d^{2}\left(\frac{(b x+a)\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}+\frac{\cos (b x+a)^{3}}{45}\right.\right.$
$\left.\left.+\frac{2 \cos (b x+a)}{15}-\frac{(b x+a)\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{\cos (b x+a)^{5}}{25}\right)\right)$
$+\frac{1}{b}\left(2 c d\left(\frac{(b x+a)\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{3}+\frac{\cos (b x+a)^{3}}{45}+\frac{2 \cos (b x+a)}{15}\right.\right.$

$$
\begin{aligned}
& \left.\left.-\frac{(b x+a)\left(\frac{8}{3}+\cos (b x+a)^{4}+\frac{4 \cos (b x+a)^{2}}{3}\right) \sin (b x+a)}{5}-\frac{\cos (b x+a)^{5}}{25}\right)\right) \\
& +\frac{a^{2} d^{2}\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)}{b^{2}} \\
& -\frac{2 a c d\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)}{b}+c^{2}\left(-\frac{\sin (b x+a) \cos (b x+a)^{4}}{5}\right. \\
& \left.\left.+\frac{\left(2+\cos (b x+a)^{2}\right) \sin (b x+a)}{15}\right)\right)
\end{aligned}
$$

Problem 45: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{4} \cos (b x+a)^{3} \sin (b x+a)^{3} \mathrm{~d} x
$$

Optimal(type 3, 213 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{9 d^{4} \cos (2 b x+2 a)}{128 b^{5}}+\frac{9 d^{2}(d x+c)^{2} \cos (2 b x+2 a)}{64 b^{3}}-\frac{3(d x+c)^{4} \cos (2 b x+2 a)}{64 b}+\frac{d^{4} \cos (6 b x+6 a)}{10368 b^{5}}-\frac{d^{2}(d x+c)^{2} \cos (6 b x+6 a)}{576 b^{3}} \\
& \quad+\frac{(d x+c)^{4} \cos (6 b x+6 a)}{192 b}-\frac{9 d^{3}(d x+c) \sin (2 b x+2 a)}{64 b^{4}}+\frac{3 d(d x+c)^{3} \sin (2 b x+2 a)}{32 b^{2}}+\frac{d^{3}(d x+c) \sin (6 b x+6 a)}{1728 b^{4}} \\
& \quad-\frac{d(d x+c)^{3} \sin (6 b x+6 a)}{288 b^{2}}
\end{aligned}
$$

Result(type ?, 2060 leaves): Display of huge result suppressed!
Problem 46: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a)^{3} \sin (b x+a)^{3} \mathrm{~d} x
$$

Optimal(type 3, 165 leaves, 10 steps):
$\frac{9 d^{2}(d x+c) \cos (2 b x+2 a)}{128 b^{3}}-\frac{3(d x+c)^{3} \cos (2 b x+2 a)}{64 b}-\frac{d^{2}(d x+c) \cos (6 b x+6 a)}{1152 b^{3}}+\frac{(d x+c)^{3} \cos (6 b x+6 a)}{192 b}-\frac{9 d^{3} \sin (2 b x+2 a)}{256 b^{4}}$

$$
+\frac{9 d(d x+c)^{2} \sin (2 b x+2 a)}{128 b^{2}}+\frac{d^{3} \sin (6 b x+6 a)}{6912 b^{4}}-\frac{d(d x+c)^{2} \sin (6 b x+6 a)}{384 b^{2}}
$$

Result(type 3, 1099 leaves):

$$
\begin{aligned}
& \frac{1}{b}\left(\frac { 1 } { b ^ { 3 } } \left(d ^ { 3 } \left(\frac{(b x+a)^{3} \sin (b x+a)^{4}}{4}-\frac{3(b x+a)^{2}\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{4}-\frac{(b x+a) \sin (b x+a)^{4}}{24}\right.\right.\right. \\
& -\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{96}-\frac{b x}{18}-\frac{a}{18}+\frac{(b x+a) \cos (b x+a)^{2}}{8}-\frac{\cos (b x+a) \sin (b x+a)}{16}+\frac{(b x+a)^{3}}{12} \\
& -\frac{(b x+a)^{3} \sin (b x+a)^{6}}{6}+\frac{(b x+a)^{2}\left(-\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{6}+\frac{5 b x}{16}+\frac{5 a}{16}\right)}{2} \\
& \left.\left.+\frac{(b x+a) \sin (b x+a)^{6}}{36}+\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{216}\right)\right)-\frac{1}{b^{3}}\left(3 a d ^ { 3 } \left(\frac{(b x+a)^{2} \sin (b x+a)^{4}}{4}\right.\right. \\
& -\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{2}+\frac{(b x+a)^{2}}{24}-\frac{\sin (b x+a)^{4}}{72}-\frac{\sin (b x+a)^{2}}{24} \\
& \left.\left.-\frac{(b x+a)^{2} \sin (b x+a)^{6}}{6}+\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{6}+\frac{5 b x}{16}+\frac{5 a}{16}\right)}{3}+\frac{\sin (b x+a)^{6}}{108}\right)\right) \\
& +\frac{1}{b^{2}}\left(3 c d ^ { 2 } \left(\frac{(b x+a)^{2} \sin (b x+a)^{4}}{4}-\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{4}+\frac{3 b x}{8}+\frac{3 a}{8}\right)}{2}+\frac{(b x+a)^{2}}{24}\right.\right. \\
& -\frac{\sin (b x+a)^{4}}{72}-\frac{\sin (b x+a)^{2}}{24}-\frac{(b x+a)^{2} \sin (b x+a)^{6}}{6}
\end{aligned}
$$

$$
\begin{aligned}
& \left.\left.+\frac{(b x+a)\left(-\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{6}+\frac{5 b x}{16}+\frac{5 a}{16}\right)}{3}+\frac{\sin (b x+a)^{6}}{108}\right)\right) \\
& +\frac{1}{b^{3}}\left(3 a ^ { 2 } d ^ { 3 } \left(\frac{(b x+a) \sin (b x+a)^{4}}{4}+\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{b x}{24}-\frac{a}{24}-\frac{(b x+a) \sin (b x+a)^{6}}{6}\right.\right. \\
& \left.\left.-\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{36}\right)\right)-\frac{1}{b^{2}}\left(6 a c d ^ { 2 } \left(\frac{(b x+a) \sin (b x+a)^{4}}{4}\right.\right. \\
& +\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{b x}{24}-\frac{a}{24}-\frac{(b x+a) \sin (b x+a)^{6}}{6} \\
& \left.\left.-\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{36}\right)\right)+\frac{1}{b}\left(3 c ^ { 2 } d \left(\frac{(b x+a) \sin (b x+a)^{4}}{4}\right.\right. \\
& +\frac{\left(\sin (b x+a)^{3}+\frac{3 \sin (b x+a)}{2}\right) \cos (b x+a)}{16}-\frac{b x}{24}-\frac{a}{24}-\frac{(b x+a) \sin (b x+a)^{6}}{6} \\
& \left.\left.-\frac{\left(\sin (b x+a)^{5}+\frac{5 \sin (b x+a)^{3}}{4}+\frac{15 \sin (b x+a)}{8}\right) \cos (b x+a)}{36}\right)\right)-\frac{a^{3} d^{3}\left(-\frac{\sin (b x+a)^{2} \cos (b x+a)^{4}}{6}-\frac{\cos (b x+a)^{4}}{12}\right)}{b^{3}} \\
& +\frac{3 a^{2} c d^{2}\left(-\frac{\sin (b x+a)^{2} \cos (b x+a)^{4}}{6}-\frac{\cos (b x+a)^{4}}{12}\right)}{b^{2}}-\frac{3 a c^{2} d\left(-\frac{\sin (b x+a)^{2} \cos (b x+a)^{4}}{6}-\frac{\cos (b x+a)^{4}}{12}\right)}{b}+c^{3}( \\
& \left.\left.-\frac{\sin (b x+a)^{2} \cos (b x+a)^{4}}{6}-\frac{\cos (b x+a)^{4}}{12}\right)\right)
\end{aligned}
$$

Problem 47: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \cos (b x+a)^{2} \cot (b x+a) \mathrm{d} x
$$

Optimal(type 4, 215 leaves, 12 steps):

$$
-\frac{3 d^{3} x}{8 b^{3}}+\frac{(d x+c)^{3}}{4 b}-\frac{\mathrm{I}(d x+c)^{4}}{4 d}+\frac{(d x+c)^{3} \ln \left(1-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}-\frac{3 \mathrm{I} d(d x+c)^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}+\frac{3 d^{2}(d x+c) \operatorname{poly} \log \left(3, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}
$$

$$
\begin{aligned}
& +\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(4, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b^{4}}+\frac{3 d^{3} \cos (b x+a) \sin (b x+a)}{8 b^{4}}-\frac{3 d(d x+c)^{2} \cos (b x+a) \sin (b x+a)}{4 b^{2}}+\frac{3 d^{2}(d x+c) \sin (b x+a)^{2}}{4 b^{3}} \\
& -\frac{(d x+c)^{3} \sin (b x+a)^{2}}{2 b}
\end{aligned}
$$

Result(type 4, 1000 leaves):
$\frac{\left(4 d^{3} x^{3} b^{3}+6 \mathrm{I} b^{2} d^{3} x^{2}+12 b^{3} c d^{2} x^{2}+12 \mathrm{I} b^{2} c d^{2} x+12 b^{3} c^{2} d x+6 \mathrm{I} c^{2} d b^{2}+4 b^{3} c^{3}-6 b d^{3} x-3 \mathrm{I} d^{3}-6 c d^{2} b\right) \mathrm{e}^{2 \mathrm{I}(b x+a)}}{32 b^{4}}$

$$
\begin{aligned}
& +\frac{\left(4 d^{3} x^{3} b^{3}-6 \mathrm{I} b^{2} d^{3} x^{2}+12 b^{3} c d^{2} x^{2}-12 \mathrm{I} b^{2} c d^{2} x+12 b^{3} c^{2} d x-6 \mathrm{I} c^{2} d b^{2}+4 b^{3} c^{3}-6 b d^{3} x+3 \mathrm{I} d^{3}-6 c d^{2} b\right) \mathrm{e}^{-2 \mathrm{I}(b x+a)}}{32 b^{4}}+\mathrm{I} c^{3} x \\
& +\frac{6 a c^{2} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{3 a c^{2} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}-\frac{6 a^{2} c d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 a^{2} c d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}-\frac{\mathrm{I} d^{3} x^{4}}{4}-\frac{2 c^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b} \\
& \left.+\frac{c^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}-\mathrm{I} c d^{2} x^{3}-\frac{3 \mathrm{I} c^{2} d x^{2}}{2}+\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c^{2} d x}{b}+\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a c^{2} d}{b^{2}}+\frac{d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right.}{b}+1\right) x^{3} \\
& b \\
& +\frac{d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{b}+\frac{d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{3}}{b^{4}}+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) c^{2} d x}{b}+\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c d^{2} x^{2}}{b}+\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) c d^{2} x^{2}}{b} \\
& -\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2} c d^{2}}{b^{3}}+\frac{6 \mathrm{I} d^{3} p o l y \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{2 a^{3} d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{a^{3} d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{4}}-\frac{3 \mathrm{I} a^{4} d^{3}}{2 b^{4}}+\frac{6 \mathrm{I} d^{3} p o l y \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& -\frac{3 \mathrm{I} c^{2} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{3 \mathrm{I} c^{2} d \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{3 \mathrm{I} c^{2} d a^{2}}{b^{2}}-\frac{3 \mathrm{I} d^{3} p o l y \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{3 \mathrm{I} d^{3} p o l y \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}} \\
& +\frac{4 \mathrm{I} c d^{2} a^{3}}{b^{3}}-\frac{2 \mathrm{I} d^{3} a^{3} x}{b^{3}}-\frac{6 \mathrm{I} c d^{2} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{6 \mathrm{I} c d^{2} a^{2} x}{b^{2}}-\frac{6 \mathrm{I} c d^{2} p o l y \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{6 \mathrm{I} c^{2} d a x}{b} \\
& +\frac{6 d^{3} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{6 d^{3} \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{6 c d^{2} p^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{6 c d^{2} p o l y \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}
\end{aligned}
$$

Problem 48: Result more than twice size of optimal antiderivative.

$$
\int(d x+c) \cos (b x+a)^{2} \cot (b x+a) \mathrm{d} x
$$

Optimal(type 4, 98 leaves, 8 steps):

$$
\frac{d x}{4 b}-\frac{\mathrm{I}(d x+c)^{2}}{2 d}+\frac{(d x+c) \ln \left(1-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}-\frac{d \cos (b x+a) \sin (b x+a)}{4 b^{2}}-\frac{(d x+c) \sin (b x+a)^{2}}{2 b}
$$

Result(type 4, 270 leaves):

$$
\begin{aligned}
& -\frac{\mathrm{I} d x^{2}}{2}-\frac{2 \mathrm{I} d a x}{b}+\frac{(2 b d x+\mathrm{I} d+2 c b) \mathrm{e}^{2 \mathrm{I}(b x+a)}}{16 b^{2}}+\frac{(2 b d x-\mathrm{I} d+2 c b) \mathrm{e}^{-2 \mathrm{I}(b x+a)}}{16 b^{2}}-\frac{2 c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b} \\
& \quad-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{\mathrm{I} d a^{2}}{b^{2}}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}+\mathrm{I} c x+\frac{d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}
\end{aligned}
$$

$$
+\frac{2 a d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{a d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}
$$

Problem 56: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \sec (b x+a) \sin (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 4, 169 leaves, 11 steps):

$$
\begin{array}{r}
-\frac{2 \mathrm{I}(d x+c)^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}-\frac{2 d(d x+c) \cos (b x+a)}{b^{2}}+\frac{2 \mathrm{I} d(d x+c) \operatorname{polylog}\left(2,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} d(d x+c) \operatorname{poly} \log \left(2, \mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}} \\
-\frac{2 d^{2} \operatorname{poly} \log \left(3,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 d^{2} \operatorname{polylog}\left(3, \mathrm{Ie} \mathrm{I}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 d^{2} \sin (b x+a)}{b^{3}}-\frac{(d x+c)^{2} \sin (b x+a)}{b}
\end{array}
$$

Result(type 4, 511 leaves):

$$
\begin{aligned}
& \frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2,-\mathrm{I}{ }^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{a^{2} d^{2} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 c d \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}+\frac{a^{2} d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& +\frac{d^{2} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x^{2}}{b}-\frac{2 d^{2} \operatorname{poly} \log \left(3,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 c d \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}+\frac{4 \mathrm{I} a c d \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} d^{2} p o l y \log \left(2, \mathrm{I} \mathrm{I}^{\mathrm{I}(b x+a)}\right) x}{b^{2}} \\
& -\frac{2 c d \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b}-\frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} a^{2} d^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 \mathrm{I} c^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{2 d^{2} p o l y \log \left(3, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& \\
& +\frac{2 c d \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}-\frac{\mathrm{I}\left(d^{2} x^{2} b^{2}+2 b^{2} c d x+b^{2} c^{2}-2 \mathrm{I} b d^{2} x-2 d^{2}-2 \mathrm{I} b c d\right) \mathrm{e}^{-\mathrm{I}(b x+a)}}{2 b^{3}}-\frac{d^{2} \ln \left(1+\mathrm{Ie} \mathrm{I}^{\mathrm{I}(b x+a)}\right) x^{2}}{b} \\
& +\frac{\mathrm{I}\left(d^{2} x^{2} b^{2}+2 b^{2} c d x+b^{2} c^{2}+2 \mathrm{I} b d^{2} x-2 d^{2}+2 \mathrm{I} b c d\right) \mathrm{e}^{\mathrm{I}(b x+a)}}{2 b^{3}}
\end{aligned}
$$

Problem 60: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{4} \csc (b x+a) \sec (b x+a) \mathrm{d} x
$$

Optimal(type 4, 221 leaves, 12 steps):
$-\frac{2(d x+c)^{4} \operatorname{arctanh}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}+\frac{2 \mathrm{I} d(d x+c)^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} d(d x+c)^{3} p o l y \log \left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}$

$$
\begin{aligned}
& -\frac{3 d^{2}(d x+c)^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 d^{2}(d x+c)^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{3 \mathrm{I} d^{3}(d x+c) \operatorname{poly} \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{4}} \\
& +\frac{3 \mathrm{I} d^{3}(d x+c) \operatorname{poly} \log \left(4, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{3 d^{4} \operatorname{poly} \log \left(5,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{5}}-\frac{3 d^{4} \operatorname{polylog}\left(5, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{5}}
\end{aligned}
$$

Result(type 4, 1241 leaves):
$\frac{4 c^{3} d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{6 a^{2} c^{2} d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}-\frac{4 a c^{3} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}-\frac{4 a^{3} c d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{4}}+\frac{d^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{4}}{b}$

$$
\begin{aligned}
& -\frac{d^{4} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{4}}{b}+\frac{d^{4} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{4}}{b}-\frac{4 c d^{3} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{3}}{b}+\frac{4 c d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{3}}{b}+\frac{4 c d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{b} \\
& +\frac{4 c d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{3}}{b^{4}}+\frac{3 d^{4} \operatorname{polylog}\left(5,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{5}}-\frac{3 \mathrm{I} c d^{3} \operatorname{polylog}\left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{24 \mathrm{I} c d^{3} \operatorname{polylog}\left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& -\frac{4 \mathrm{I} d^{4} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{b^{2}}-\frac{4 \mathrm{I} d^{4} \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{b^{2}}-\frac{4 \mathrm{I} c^{3} d \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{2 \mathrm{I} c^{3} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}} \\
& -\frac{4 \mathrm{I} c^{3} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{4}}+\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{4}}+\frac{2 \mathrm{I} d^{4} \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{3}}{b^{2}} \\
& -\frac{3 \mathrm{I} d^{4} \text { poly } \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{4}}+\frac{24 \mathrm{I} c d^{3} \operatorname{poly} \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{6 \mathrm{I} c^{2} d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{12 \mathrm{I} c^{2} d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}} \\
& +\frac{6 \mathrm{I} c d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{12 \mathrm{I} c d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{12 \mathrm{I} c d^{3} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{12 \mathrm{I}^{2} d^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}} \\
& -\frac{d^{4} a^{4} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}+\frac{12 d^{4} \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{3}}+\frac{12 d^{4} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{3}}-\frac{3 d^{4} \operatorname{polylog}\left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{2}}{b^{3}}+\frac{a^{4} d^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{5}} \\
& +\frac{12 c^{2} d^{2} \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{3 c^{2} d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{12 c^{2} d^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{24 d^{4} \operatorname{polylog}\left(5, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}} \\
& -\frac{24 d^{4} \operatorname{polylog}\left(5,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}-\frac{c^{4} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}+\frac{c^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}+\frac{4 c^{3} d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}} \\
& +\frac{6 c^{2} d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{2}}{b}-\frac{6 c d^{3} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{4 c^{3} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}-\frac{6 c^{2} d^{2} a^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& +\frac{24 c d^{3} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{6 c^{2} d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b}-\frac{6 c^{2} d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{2}}{b}-\frac{4 c^{3} d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b} \\
& +\frac{24 c d^{3} \text { polylog }\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}
\end{aligned}
$$

Problem 61: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \csc (b x+a) \sec (b x+a) \mathrm{d} x
$$

Optimal(type 4, 111 leaves, 8 steps):
$-\frac{2(d x+c)^{2} \operatorname{arctanh}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}+\frac{\mathrm{I} d(d x+c) \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{\mathrm{I} d(d x+c) \operatorname{polylog}\left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}$
$+\frac{d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}$
Result(type 4, 468 leaves):
$-\frac{d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{2 c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}+\frac{\mathrm{I} d^{2} p o l y \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{2}}$

$$
\begin{aligned}
& -\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{a^{2} d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}-\frac{d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{2}}{b}+\frac{2 d^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& +\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b}-\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2}}{b^{3}}+\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{2}}{b}-\frac{2 c d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}-\frac{2 a c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}} \\
& -\frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{c^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}+\frac{c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b} \\
& -\frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{\mathrm{I} c d \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}
\end{aligned}
$$

Problem 62: Result more than twice size of optimal antiderivative.

$$
\int(d x+c) \csc (b x+a) \sec (b x+a) \mathrm{d} x
$$

Optimal(type 4, 59 leaves, 6 steps):

$$
-\frac{2(d x+c) \operatorname{arctanh}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}+\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}
$$

Result(type 4, 207 leaves):

$$
\begin{array}{r}
-\frac{c \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}} \\
-\frac{d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}+\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}+\frac{d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{a d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}
\end{array}
$$

Problem 66: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \csc (b x+a)^{3} \sec (b x+a) \mathrm{d} x
$$

Optimal(type 4, 181 leaves, 17 steps):

$$
\begin{aligned}
& -\frac{c d x}{b}-\frac{d^{2} x^{2}}{2 b}-\frac{2(d x+c)^{2} \operatorname{arctanh}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}-\frac{d(d x+c) \cot (b x+a)}{b^{2}}-\frac{(d x+c)^{2} \cot (b x+a)^{2}}{2 b}+\frac{d^{2} \ln (\sin (b x+a))}{b^{3}} \\
& \quad+\frac{\mathrm{I} d(d x+c) \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{\mathrm{I} d(d x+c) \operatorname{poly} \log \left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}
\end{aligned}
$$

## Result(type 4, 631 leaves):

$$
\begin{array}{r}
-\frac{d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{2 c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}-\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}} \\
-\frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{a^{2} d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}-\frac{d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{2}}{b}+\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b^{3}}-\frac{2 d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
\quad+\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}+\frac{2 d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b}-\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2}}{b^{3}}
\end{array}
$$

$$
\begin{aligned}
& +\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{2}}{b}-\frac{2 c d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}-\frac{2 a c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}+\frac{\mathrm{I} c d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{\mathrm{I} d^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{2}} \\
& +\frac{2\left(b d^{2} x^{2} \mathrm{e}^{2 \mathrm{I}(b x+a)}+2 b c d x \mathrm{e}^{2 \mathrm{I}(b x+a)}+b c^{2} \mathrm{e}^{2 \mathrm{I}(b x+a)}-\mathrm{I} d^{2} x \mathrm{e}^{2 \mathrm{I}(b x+a)}-\mathrm{I} c d \mathrm{e}^{2 \mathrm{I}(b x+a)}+\mathrm{I} d^{2} x+\mathrm{I} c d\right)}{b^{2}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)^{2}}-\frac{c^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b} \\
& +\frac{c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}-\frac{2 \mathrm{I} c d \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}
\end{aligned}
$$

Problem 67: Result more than twice size of optimal antiderivative.

$$
\int(d x+c) \csc (b x+a)^{3} \sec (b x+a) \mathrm{d} x
$$

Optimal(type 4, 123 leaves, 11 steps):

$$
\begin{aligned}
-\frac{d x}{2 b} & -\frac{2 d x \operatorname{arctanh}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}-\frac{d \cot (b x+a)}{2 b^{2}}-\frac{(d x+c) \cot (b x+a)^{2}}{2 b}-\frac{d x \ln (\tan (b x+a))}{b}+\frac{(d x+c) \ln (\tan (b x+a))}{b} \\
& +\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Result (type 4, } 269 \text { leaves) : } \\
& \frac{2 \mathrm{e}^{2 \mathrm{I}(b x+a)} b d x-\mathrm{I} d \mathrm{e}^{2 \mathrm{I}(b x+a)}+2 \mathrm{e}^{2 \mathrm{I}(b x+a)} b c+\mathrm{I} d}{b^{2}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)^{2}}-\frac{c \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b} \\
& \quad+\frac{d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}-\frac{\mathrm{I} d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}+\frac{\mathrm{I} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}+\frac{d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b} \\
& \quad-\frac{\mathrm{I} d \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{a d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}
\end{aligned}
$$

Problem 68: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{4} \sec (b x+a) \tan (b x+a) \mathrm{d} x
$$

Optimal(type 4, 202 leaves, 10 steps):

$$
\begin{aligned}
& \frac{8 \mathrm{I} d(d x+c)^{3} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{12 \mathrm{I} d^{2}(d x+c)^{2} \operatorname{poly} \log \left(2,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{12 \mathrm{I} d^{2}(d x+c)^{2} \operatorname{poly} \log \left(2, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& \quad+\frac{24 d^{3}(d x+c) \operatorname{poly} \log \left(3,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{24 d^{3}(d x+c) \operatorname{poly} \log \left(3, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{5}}-\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{5}} \\
& \quad+\frac{(d x+c)^{4} \sec (b x+a)}{b}
\end{aligned}
$$

Result(type 4, 766 leaves):
$-\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}-\frac{24 \mathrm{I} d^{3} c \operatorname{poly} \log \left(2,-\mathrm{I} \mathrm{I}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{24 \mathrm{I} d^{3} c \operatorname{poly} \log \left(2, \mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{24 \mathrm{I} d^{3} a^{2} c \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}$

$$
\begin{aligned}
& -\frac{24 \mathrm{I} d^{2} a c^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{24 \mathrm{I} d^{4} \operatorname{poly} \log \left(4,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}+\frac{2 \mathrm{e}^{\mathrm{I}(b x+a)}\left(d^{4} x^{4}+4 c d^{3} x^{3}+6 c^{2} d^{2} x^{2}+4 c^{3} d x+c^{4}\right)}{\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) b} \\
& -\frac{24 d^{4} \operatorname{poly} \log \left(3, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{4}}-\frac{4 d^{4} a^{3} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}+\frac{24 d^{3} c \operatorname{poly} \log \left(3,-\mathrm{I}{ }^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{24 d^{4} \operatorname{poly} \log \left(3,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{4}} \\
& +\frac{4 d^{4} a^{3} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{5}}+\frac{4 d^{4} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x^{3}}{b^{2}}-\frac{4 d^{4} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x^{3}}{b^{2}}-\frac{24 d^{3} c \operatorname{poly} \log \left(3, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{12 d^{2} c^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{2}} \\
& +\frac{12 d^{2} c^{2} \ln \left(1+\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}-\frac{12 d^{2} c^{2} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{12 d^{2} c^{2} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}-\frac{12 d^{3} a^{2} c \ln \left(1+\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& +\frac{12 d^{3} a^{2} c \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{12 d^{3} c \ln \left(1+\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{12 d^{3} c \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{12 \mathrm{I} d^{2} c^{2} \operatorname{polylog}\left(2,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& +\frac{12 \mathrm{I} d^{2} c^{2} \operatorname{polylog}\left(2, \mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{8 \mathrm{I} d c^{3} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{8 \mathrm{I} d^{4} a^{3} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}-\frac{12 \mathrm{I} d^{4} \operatorname{polylog}\left(2,-\mathrm{I} \mathrm{I}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{3}} \\
& +\frac{12 \mathrm{I} d^{4} \operatorname{poly} \log \left(2, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{3}}
\end{aligned}
$$

Problem 73: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \csc (b x+a) \sec (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 4, 308 leaves, 23 steps):
$\frac{6 \mathrm{I} d(d x+c)^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2(d x+c)^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{3 \mathrm{I} d(d x+c)^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{6 \mathrm{I} d^{2}(d x+c) \operatorname{poly} \log \left(2,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}$

$$
\begin{aligned}
& +\frac{6 \mathrm{I} d^{2}(d x+c) \operatorname{poly} \log \left(2, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{3 \mathrm{I} d(d x+c)^{2} \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{6 d^{2}(d x+c) \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{6 d^{3} p o l y \log \left(3,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& -\frac{6 d^{3} \operatorname{poly} \log \left(3, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{6 d^{2}(d x+c) \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{6 \mathrm{I} d^{3} \operatorname{poly} \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{6 \mathrm{I} d^{3} \operatorname{poly} \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& +\frac{(d x+c)^{3} \sec (b x+a)}{b}
\end{aligned}
$$

Result(type 4, 1151 leaves):

$$
\begin{aligned}
& -\frac{3 a c^{2} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}+\frac{3 a^{2} c d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}+\frac{c^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}-\frac{c^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}-\frac{6 \mathrm{I} d^{3} \operatorname{poly} \log \left(4,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& -\frac{12 \mathrm{I} c d^{2} a \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{6 \mathrm{I} c d^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{6 c d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}-\frac{6 c d^{2} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{3}} \\
& +\frac{6 c d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{6 c d^{2} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{3 \mathrm{I} c^{2} d \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{6 \mathrm{I} d^{3} a^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}+\frac{6 \mathrm{I} d^{3} \operatorname{polylog}\left(2, \mathrm{Ie} \mathrm{I}^{\mathrm{I}(b x+a)}\right) x}{b^{3}} \\
& \\
& +\frac{6 \mathrm{I} d^{3} \operatorname{poly} \log \left(2, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{4}}-\frac{6 \mathrm{I} d^{3} \operatorname{poly} \log \left(2,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}-\frac{6 \mathrm{I} d^{3} \operatorname{poly} \log \left(2,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a}{b^{4}}+\frac{6 \mathrm{I} c^{2} d \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{6 \mathrm{I} a d^{3} \operatorname{dilog}\left(1+\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{6 \mathrm{I} a d^{3} \operatorname{dilog}\left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{6 \mathrm{I} c d^{2} \operatorname{dilog}\left(1+\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{6 \mathrm{I} c d^{2} \operatorname{dilog}\left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& +\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}+\frac{3 d^{3} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2}}{b^{4}}-\frac{3 d^{3} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a^{2}}{b^{4}}-\frac{3 d^{3} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}+\frac{3 d^{3} \ln \left(1+\mathrm{I}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}} \\
& +\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c^{2} d x}{b}+\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a c^{2} d}{b^{2}}-\frac{d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{3}}{b}+\frac{d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{3}}{b}+\frac{d^{3} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{3}}{b^{4}} \\
& -\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) c^{2} d x}{b}+\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) c d^{2} x^{2}}{b}-\frac{3 \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) c d^{2} x^{2}}{b}-\frac{3 \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2} c d^{2}}{b^{3}}+\frac{6 \mathrm{I} d^{3} p o l y \log \left(4, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}} \\
& +\frac{2 \mathrm{e}^{\mathrm{I}(b x+a)}\left(d^{3} x^{3}+3 c d^{2} x^{2}+3 c^{2} d x+c^{3}\right)}{\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}-\frac{a^{3} d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{4}}-\frac{3 \mathrm{I} c^{2} d \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{3 \mathrm{I} d^{3} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b^{2}} \\
& \\
& -\frac{6 \mathrm{I} c d^{2} \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{6 d^{3} p o l y \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{6 d^{3} p o l y \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{6 c d^{2} p o l y \log \left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& -\frac{6 c d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{6 d^{3} p o l y \log \left(3,-\mathrm{I}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{6 d^{3} \operatorname{poly\operatorname {log}(3,\mathrm {I}\mathrm {e}^{\mathrm {I}(bx+a)})}}{b^{4}}
\end{aligned}
$$

Problem 80: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \sec (b x+a) \tan (b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 4, 174 leaves, 17 steps):
$\frac{\mathrm{I}(d x+c)^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{d^{2} \operatorname{arctanh}(\sin (b x+a))}{b^{3}}-\frac{\mathrm{I} d(d x+c) \operatorname{poly} \log \left(2,-\mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{\mathrm{I} d(d x+c) \operatorname{polylog}\left(2, \mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}$
$+\frac{d^{2} \operatorname{poly} \log \left(3,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{d^{2} \operatorname{poly} \log \left(3, \mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{d(d x+c) \sec (b x+a)}{b^{2}}+\frac{(d x+c)^{2} \sec (b x+a) \tan (b x+a)}{2 b}$
Result(type 4, 583 leaves):

$$
\begin{aligned}
& -\frac{\mathrm{I} c d \operatorname{poly} \log \left(2,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d^{2} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x^{2}}{2 b}-\frac{a^{2} d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{d^{2} \operatorname{poly} \log \left(3,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{a^{2} d^{2} \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{2 b^{3}} \\
& -\frac{2 \mathrm{I} d^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{\mathrm{I} d^{2} \operatorname{polylog}\left(2,-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{2 \mathrm{I} a c d \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{\mathrm{I} d^{2} p o l y \log \left(2, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}+\frac{\mathrm{I} c d \operatorname{poly} \log \left(2, \mathrm{Ie} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}} \\
& -\frac{c d \ln \left(1-\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b}-\frac{1}{\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)^{2} b^{2}}\left(\mathrm { I } \left(x^{2} d^{2} b \mathrm{e}^{3 \mathrm{I}(b x+a)}+2 c d x b \mathrm{e}^{3 \mathrm{I}(b x+a)}+c^{2} b \mathrm{e}^{3 \mathrm{I}(b x+a)}-x^{2} d^{2} b \mathrm{e}^{\mathrm{I}(b x+a)}-2 c d x b \mathrm{e}^{\mathrm{I}(b x+a)}\right.\right. \\
& \\
& \left.\left.-2 \mathrm{I} d^{2} x \mathrm{e}^{3 \mathrm{I}(b x+a)}-c^{2} b \mathrm{e}^{\mathrm{I}(b x+a)}-2 \mathrm{I} c d \mathrm{e}^{3 \mathrm{I}(b x+a)}-2 \mathrm{I}^{2} x \mathrm{e}^{\mathrm{I}(b x+a)}-2 \mathrm{I} c d \mathrm{e}^{\mathrm{I}(b x+a)}\right)\right)+\frac{\mathrm{I} c^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}+\frac{\mathrm{I} a^{2} d^{2} \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}} \\
& \\
& +\frac{c d \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}-\frac{c d \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}+\frac{d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x^{2}}{2 b}-\frac{d^{2} \operatorname{polylog}\left(3, \mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{c d \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b}
\end{aligned}
$$

Problem 83: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \csc (b x+a) \sec (b x+a)^{3} \mathrm{~d} x
$$

Optimal(type 4, 181 leaves, 17 steps):

$$
\begin{aligned}
\frac{c d x}{b} & +\frac{d^{2} x^{2}}{2 b}-\frac{2(d x+c)^{2} \operatorname{arctanh}\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b}-\frac{d^{2} \ln (\cos (b x+a))}{b^{3}}+\frac{\mathrm{I} d(d x+c) \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{\mathrm{I} d(d x+c) \operatorname{polylog}\left(2, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}} \\
& -\frac{d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{d^{2} \operatorname{poly} \log \left(3, \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}-\frac{d(d x+c) \tan (b x+a)}{b^{2}}+\frac{(d x+c)^{2} \tan (b x+a)^{2}}{2 b}
\end{aligned}
$$

Result(type 4, 613 leaves):

```
\(-\frac{d^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{2 c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b}+\frac{2 c d \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{2}}-\frac{2 \mathrm{I} d^{2} p o l y \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}\)
    \(-\frac{2 \mathrm{I} c d \operatorname{poly} \log \left(2, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b^{3}}+\frac{a^{2} d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{3}}-\frac{d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{2}}{b}+\frac{2 d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}\)
    \(+\frac{2 d^{2} \operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{2 d^{2} \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x^{2}}{b}-\frac{d^{2} \ln \left(1-\mathrm{e}^{\mathrm{I}(b x+a)}\right) a^{2}}{b^{3}}+\frac{d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right) x^{2}}{b}\)
    \(-\frac{2 c d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}-\frac{2 a c d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b^{2}}+\frac{\mathrm{I} c d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{\mathrm{I} d^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{c^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}\)
    \(+\frac{c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}-1\right)}{b}+\frac{c^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}+1\right)}{b}\)
    \(+\frac{2\left(b d^{2} x^{2} \mathrm{e}^{2 \mathrm{I}(b x+a)}+2 b c d x \mathrm{e}^{2 \mathrm{I}(b x+a)}+b c^{2} \mathrm{e}^{2 \mathrm{I}(b x+a)}-\mathrm{I} d^{2} x \mathrm{e}^{2 \mathrm{I}(b x+a)}-\mathrm{I} c d \mathrm{e}^{2 \mathrm{I}(b x+a)}-\mathrm{I} d^{2} x-\mathrm{I} c d\right)}{\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)^{2} b^{2}}-\frac{2 \mathrm{I} c d \mathrm{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}\)
    \(-\frac{2 \mathrm{I} d^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}\)
```

Problem 86: Unable to integrate problem.

$$
\int x \sin (b x+a) \sqrt{\cos (b x+a)} \mathrm{d} x
$$

Optimal (type 4, 76 leaves, 3 steps):

$$
-\frac{2 x \cos (b x+a)^{3 / 2}}{3 b}+\frac{4 \sqrt{\cos \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{b x}{2}+\frac{a}{2}\right), \sqrt{2}\right)}{9 \cos \left(\frac{b x}{2}+\frac{a}{2}\right) b^{2}}+\frac{4 \sin (b x+a) \sqrt{\cos (b x+a)}}{9 b^{2}}
$$

Result(type 8, 18 leaves):

$$
\int x \sin (b x+a) \sqrt{\cos (b x+a)} \mathrm{d} x
$$

Problem 87: Unable to integrate problem.

$$
\int \frac{x \sin (b x+a)}{\cos (b x+a)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 57 leaves, 2 steps):

$$
-\frac{4 \sqrt{\cos \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{b x}{2}+\frac{a}{2}\right), \sqrt{2}\right)}{\cos \left(\frac{b x}{2}+\frac{a}{2}\right) b^{2}}+\frac{2 x}{b \sqrt{\cos (b x+a)}}
$$

Result(type 8, 18 leaves):

$$
\int \frac{x \sin (b x+a)}{\cos (b x+a)^{3 / 2}} \mathrm{~d} x
$$

Problem 88: Unable to integrate problem.

$$
\int \frac{x \sin (b x+a)}{\cos (b x+a)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 76 leaves, 3 steps):

$$
\frac{2 x}{3 b \cos (b x+a)^{3 / 2}}+\frac{4 \sqrt{\cos \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{b x}{2}+\frac{a}{2}\right), \sqrt{2}\right)}{3 \cos \left(\frac{b x}{2}+\frac{a}{2}\right) b^{2}}-\frac{4 \sin (b x+a)}{3 b^{2} \sqrt{\cos (b x+a)}}
$$

Result(type 8, 18 leaves):

$$
\int \frac{x \sin (b x+a)}{\cos (b x+a)^{5 / 2}} \mathrm{~d} x
$$

Problem 89: Unable to integrate problem.

$$
\int \frac{x \sin (b x+a)}{\cos (b x+a)^{9 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 95 leaves, 4 steps):

$$
\frac{2 x}{7 b \cos (b x+a)^{7 / 2}}+\frac{12 \sqrt{\cos \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{b x}{2}+\frac{a}{2}\right), \sqrt{2}\right)}{35 \cos \left(\frac{b x}{2}+\frac{a}{2}\right) b^{2}}-\frac{4 \sin (b x+a)}{35 b^{2} \cos (b x+a)^{5 / 2}}-\frac{12 \sin (b x+a)}{35 b^{2} \sqrt{\cos (b x+a)}}
$$

Result(type 8, 18 leaves):

$$
\int \frac{x \sin (b x+a)}{\cos (b x+a)^{9 / 2}} \mathrm{~d} x
$$

Problem 90: Unable to integrate problem.

$$
\int x \sec (b x+a)^{7 / 2} \sin (b x+a) \mathrm{d} x
$$

Optimal(type 4, 92 leaves, 4 steps):

$$
\frac{2 x \sec (b x+a)^{5 / 2}}{5 b}-\frac{4 \sec (b x+a)^{3 / 2} \sin (b x+a)}{15 b^{2}}-\frac{4 \sqrt{\cos \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{b x}{2}+\frac{a}{2}\right), \sqrt{2}\right) \sqrt{\cos (b x+a)} \sqrt{\sec (b x+a)}}{15 \cos \left(\frac{b x}{2}+\frac{a}{2}\right) b^{2}}
$$

Result(type 8, 18 leaves):

$$
\int x \sec (b x+a)^{7 / 2} \sin (b x+a) \mathrm{d} x
$$

Problem 91: Unable to integrate problem.

$$
\int x \sec (b x+a)^{3 / 2} \sin (b x+a) d x
$$

Optimal(type 4, 73 leaves, 3 steps):


Result(type 8, 18 leaves):

$$
\int x \sec (b x+a)^{3 / 2} \sin (b x+a) \mathrm{d} x
$$

Problem 92: Unable to integrate problem.

$$
\int \frac{x \sin (b x+a)}{\sec (b x+a)^{5 / 2}} d x
$$

Optimal (type 4, 111 leaves, 5 steps):
$-\frac{2 x}{7 b \sec (b x+a)^{7 / 2}}+\frac{4 \sin (b x+a)}{49 b^{2} \sec (b x+a)^{5 / 2}}+\frac{20 \sin (b x+a)}{147 b^{2} \sqrt{\sec (b x+a)}}$

$$
+\frac{20 \sqrt{\cos \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{b x}{2}+\frac{a}{2}\right), \sqrt{2}\right) \sqrt{\cos (b x+a)} \sqrt{\sec (b x+a)}}{147 \cos \left(\frac{b x}{2}+\frac{a}{2}\right) b^{2}}
$$

Result (type 8, 18 leaves):

$$
\int \frac{x \sin (b x+a)}{\sec (b x+a)^{5 / 2}} d x
$$

Problem 93: Unable to integrate problem.

$$
\int x \cos (b x+a) \sin (b x+a)^{3 / 2} \mathrm{~d} x
$$

Optimal (type 4, 85 leaves, 3 steps):

$$
\frac{12 \sqrt{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}} \text { EllipticE }\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right)}{25 \sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b^{2}}+\frac{4 \cos (b x+a) \sin (b x+a)^{3 / 2}}{25 b^{2}}+\frac{2 x \sin (b x+a)^{5 / 2}}{5 b}
$$

Result(type 8, 18 leaves):

$$
\int x \cos (b x+a) \sin (b x+a)^{3 / 2} \mathrm{~d} x
$$

Problem 94: Result more than twice size of optimal antiderivative.

$$
\int \frac{x \cos (b x+a)}{\sqrt{\sin (b x+a)}} d x
$$

Optimal(type 4, 66 leaves, 2 steps):

$$
\frac{4 \sqrt{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}} \text { EllipticE }\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right)}{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b^{2}}+\frac{2 x \sqrt{\sin (b x+a)}}{b}
$$

Result(type 4, 307 leaves):

$$
\begin{aligned}
& -\frac{\mathrm{I}(b x+2 \mathrm{I})\left(\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{2}-1\right) \sqrt{2}}{b^{2} \sqrt{\frac{-\mathrm{I}\left(\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{2}-1\right)}{\mathrm{e}^{\mathrm{I}(b x+a)}}} \mathrm{e}^{\mathrm{I}(b x+a)}}-\frac{1}{b^{2} \sqrt{\frac{-\mathrm{I}\left(\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{2}-1\right)}{\mathrm{e}^{\mathrm{I}(b x+a)}}} \mathrm{e}^{\mathrm{I}(b x+a)}}\left(2 \left(\frac{2 \mathrm{I}\left(\mathrm{I}-\mathrm{I}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{2}\right)}{\sqrt{\mathrm{e}^{\mathrm{I}(b x+a)}\left(\mathrm{I}-\mathrm{I}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{2}\right)}}\right.\right. \\
& \left.-\frac{\sqrt{\mathrm{e}^{\mathrm{I}(b x+a)}+1} \sqrt{-2 \mathrm{e}^{\mathrm{I}(b x+a)}+2} \sqrt{-\mathrm{e}^{\mathrm{I}(b x+a)}}\left(-2 \operatorname{EllipticE}\left(\sqrt{\mathrm{e}^{\mathrm{I}(b x+a)}+1}, \frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\sqrt{\mathrm{e}^{\mathrm{I}(b x+a)}+1}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{-\mathrm{I}\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{3}+\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}}}\right) \\
& \left.\sqrt{2} \sqrt{-\mathrm{I}\left(\left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)^{2}-1\right) \mathrm{e}^{\mathrm{I}(b x+a)}}\right)
\end{aligned}
$$

Problem 95: Unable to integrate problem.

$$
\int \frac{x \cos (b x+a)}{\sin (b x+a)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 66 leaves, 2 steps):

$$
-\frac{4 \sqrt{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}} \text { EllipticF }\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right)}{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b^{2}}-\frac{2 x}{b \sqrt{\sin (b x+a)}}
$$

Result(type 8, 18 leaves):

$$
\int \frac{x \cos (b x+a)}{\sin (b x+a)^{3 / 2}} \mathrm{~d} x
$$

Problem 96: Unable to integrate problem.

$$
\int \frac{x \cos (b x+a)}{\sin (b x+a)^{5 / 2}} d x
$$

Optimal(type 4, 85 leaves, 3 steps):

$$
\frac{4 \sqrt{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}} \text { EllipticE }\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right)}{3 \sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b^{2}}-\frac{2 x}{3 b \sin (b x+a)^{3 / 2}}-\frac{4 \cos (b x+a)}{3 b^{2} \sqrt{\sin (b x+a)}}
$$

Result (type 8, 18 leaves):

$$
\int \frac{x \cos (b x+a)}{\sin (b x+a)^{5 / 2}} \mathrm{~d} x
$$

Problem 97: Unable to integrate problem.

$$
\int x \cos (b x+a) \csc (b x+a)^{5 / 2} \mathrm{~d} x
$$

Optimal(type 4, 101 leaves, 4 steps):
$-\frac{2 x \csc (b x+a)^{3 / 2}}{3 b}-\frac{4 \cos (b x+a) \sqrt{\csc (b x+a)}}{3 b^{2}}+\frac{4 \sqrt{\sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}} \operatorname{EllipticE}\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right) \sqrt{\csc (b x+a)} \sqrt{\sin (b x+a)}}{3 \sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b^{2}}$
Result(type 8, 18 leaves):

$$
\int x \cos (b x+a) \csc (b x+a)^{5 / 2} \mathrm{~d} x
$$

Problem 100: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{3} \csc (b x+a) \sin (3 b x+3 a) \mathrm{d} x
$$

```
Optimal(type 3, 157 leaves, 10 steps):
\[
\begin{aligned}
& -\frac{3 c d^{2} x}{2 b^{2}}-\frac{3 d^{3} x^{2}}{4 b^{2}}+\frac{(d x+c)^{4}}{4 d}-\frac{9 d^{3} \cos (b x+a)^{2}}{8 b^{4}}+\frac{9 d(d x+c)^{2} \cos (b x+a)^{2}}{4 b^{2}}-\frac{3 d^{2}(d x+c) \cos (b x+a) \sin (b x+a)}{b^{3}} \\
& +\frac{2(d x+c)^{3} \cos (b x+a) \sin (b x+a)}{b}+\frac{3 d^{3} \sin (b x+a)^{2}}{8 b^{4}}-\frac{3 d(d x+c)^{2} \sin (b x+a)^{2}}{4 b^{2}}
\end{aligned}
\]
```

Result (type 3, 579 leaves):

$$
\begin{aligned}
-c^{3} x- & \frac{d^{3} x^{4}}{4}+\frac{4 c^{3}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{b}-\frac{3 c^{2} d x^{2}}{2}-c d^{2} x^{3}+\frac{1}{b^{4}}\left(4 d ^ { 3 } \left((b x+a)^{3}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right.\right. \\
& +\frac{3(b x+a)^{2} \cos (b x+a)^{2}}{4}-\frac{3(b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{2}+\frac{3(b x+a)^{2}}{8}+\frac{3 \sin (b x+a)^{2}}{8}-\frac{3(b x+a)^{4}}{8}
\end{aligned}
$$

$$
-3 a\left((b x+a)^{2}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)+\frac{(b x+a) \cos (b x+a)^{2}}{2}-\frac{\cos (b x+a) \sin (b x+a)}{4}-\frac{b x}{4}-\frac{a}{4}-\frac{(b x+a)^{3}}{3}\right)
$$

$$
\left.\left.+3 a^{2}\left((b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{4}-\frac{\sin (b x+a)^{2}}{4}\right)-a^{3}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right)\right)
$$

$$
+\frac{12 c^{2} d\left((b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{4}-\frac{\sin (b x+a)^{2}}{4}-a\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)\right)}{b^{2}}
$$

$$
+\frac{1}{b^{3}}\left(1 2 c d ^ { 2 } \left((b x+a)^{2}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)+\frac{(b x+a) \cos (b x+a)^{2}}{2}-\frac{\cos (b x+a) \sin (b x+a)}{4}-\frac{b x}{4}-\frac{a}{4}\right.\right.
$$

$$
-\frac{(b x+a)^{3}}{3}-2 a\left((b x+a)\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{(b x+a)^{2}}{4}-\frac{\sin (b x+a)^{2}}{4}\right)+a^{2}\left(\frac{\cos (b x+a) \sin (b x+a)}{2}+\frac{b x}{2}\right.
$$

$$
\left.\left.+\frac{a}{2}\right) \int\right)
$$

Problem 103: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{4} \sec (b x+a) \sin (3 b x+3 a) \mathrm{d} x
$$

Optimal(type 4, 282 leaves, 20 steps):
$\frac{6 c d^{3} x}{b^{3}}+\frac{3 d^{4} x^{2}}{b^{3}}-\frac{(d x+c)^{4}}{b}-\frac{\mathrm{I}(d x+c)^{5}}{5 d}+\frac{(d x+c)^{4} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}-\frac{2 \mathrm{I} d(d x+c)^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}$
$+\frac{3 d^{2}(d x+c)^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{3 \mathrm{I} d^{3}(d x+c) \operatorname{poly} \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 d^{4} \operatorname{poly} \log \left(5,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{5}}$
$\quad-\frac{6 d^{3}(d x+c) \cos (b x+a) \sin (b x+a)}{b^{4}}+\frac{4 d(d x+c)^{3} \cos (b x+a) \sin (b x+a)}{b^{2}}+\frac{3 d^{4} \sin (b x+a)^{2}}{b^{5}}-\frac{6 d^{2}(d x+c)^{2} \sin (b x+a)^{2}}{b^{3}}$

$$
+\frac{2(d x+c)^{4} \sin (b x+a)^{2}}{b}
$$

Result(type 4, 955 leaves):

```
\(\frac{d^{4} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{4}}{b}+\frac{4 c d^{3} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{3}}{b}-\frac{3 d^{4} \operatorname{poly} \log \left(5,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{5}}-\mathrm{I} c d^{3} x^{4}-2 \mathrm{I} c^{2} d^{2} x^{3}-2 \mathrm{I} c^{3} d x^{2}-\frac{\mathrm{I} d^{4} x^{5}}{5}+\frac{8 a^{3} c d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}\)
```

    \(-\frac{12 a^{2} c^{2} d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{8 a c^{3} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{4 \mathrm{I} c^{3} d a^{2}}{b^{2}}-\frac{6 \mathrm{I} a^{4} c d^{3}}{b^{4}}+\frac{8 \mathrm{I} c^{2} d^{2} a^{3}}{b^{3}}+\frac{2 \mathrm{I} a^{4} d^{4} x}{b^{4}}+\frac{3 \mathrm{I} c d^{3} \operatorname{poly} \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{4}}\)
    \(-\frac{2 \mathrm{I} c^{3} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 \mathrm{I} d^{4} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{3}}{b^{2}}+\frac{3 \mathrm{I} d^{4} \operatorname{poly} \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{4}}-\frac{6 \mathrm{I} c^{2} d^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{2}}\)
    \(-\frac{6 \mathrm{I} c d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{2}}{b^{2}}-\frac{8 \mathrm{I} c^{3} d a x}{b}+\frac{12 \mathrm{I} c^{2} d^{2} a^{2} x}{b^{2}}-\frac{8 \mathrm{I} c d^{3} a^{3} x}{b^{3}}+\frac{3 d^{4} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{2}}{b^{3}}+\frac{3 c^{2} d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b^{3}}\)
    \(+\mathrm{I} c^{4} x+\frac{c^{4} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}-\frac{2 a^{4} d^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{5}}+\frac{8 \mathrm{I} d^{4} a^{5}}{5 b^{5}}-\frac{1}{4 b^{5}}\left(\left(2 b^{4} d^{4} x^{4}+4 \mathrm{I} b^{3} d^{4} x^{3}+8 b^{4} c d^{3} x^{3}+12 \mathrm{I} b^{3} c d^{3} x^{2}+12 b^{4} c^{2} d^{2} x^{2}\right.\right.\)
    \(\left.\left.+12 \mathrm{I} b^{3} c^{2} d^{2} x+8 b^{4} c^{3} d x+4 \mathrm{I} b^{3} c^{3} d+2 b^{4} c^{4}-6 b^{2} d^{4} x^{2}-6 \mathrm{I} b d^{4} x-12 b^{2} c d^{3} x-6 \mathrm{I} b c d^{3}-6 c^{2} d^{2} b^{2}+3 d^{4}\right) \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)-\frac{1}{4 b^{5}}\left(\left(2 b^{4} d^{4} x^{4}\right.\right.\)
    \(-4 \mathrm{I} b^{3} d^{4} x^{3}+8 b^{4} c d^{3} x^{3}-12 \mathrm{I} b^{3} c d^{3} x^{2}+12 b^{4} c^{2} d^{2} x^{2}-12 \mathrm{I} b^{3} c^{2} d^{2} x+8 b^{4} c^{3} d x-4 \mathrm{I} b^{3} c^{3} d+2 b^{4} c^{4}-6 b^{2} d^{4} x^{2}+6 \mathrm{I} b d^{4} x-12 b^{2} c d^{3} x+6 \mathrm{I} b c d^{3}\)
    \(\left.\left.-6 c^{2} d^{2} b^{2}+3 d^{4}\right) \mathrm{e}^{-2 \mathrm{I}(b x+a)}\right)+\frac{6 c d^{3} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{3}}+\frac{6 c^{2} d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{2}}{b}+\frac{4 c^{3} d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}-\frac{2 c^{4} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}\)
    Problem 104: Result more than twice size of optimal antiderivative.
$\int(d x+c)^{3} \sec (b x+a) \sin (3 b x+3 a) d x$
Optimal(type 4, 219 leaves, 19 steps):

$$
\begin{aligned}
& \frac{3 d^{3} x}{2 b^{3}}-\frac{(d x+c)^{3}}{b}-\frac{\mathrm{I}(d x+c)^{4}}{4 d}+\frac{(d x+c)^{3} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b}-\frac{3 \mathrm{I} d(d x+c)^{2} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}+\frac{3 d^{2}(d x+c) \operatorname{polylog}\left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}} \\
& \quad+\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b^{4}}-\frac{3 d^{3} \cos (b x+a) \sin (b x+a)}{2 b^{4}}+\frac{3 d(d x+c)^{2} \cos (b x+a) \sin (b x+a)}{b^{2}}-\frac{3 d^{2}(d x+c) \sin (b x+a)^{2}}{b^{3}} \\
& \quad+\frac{2(d x+c)^{3} \sin (b x+a)^{2}}{b}
\end{aligned}
$$

Result(type 4, 638 leaves):
$-\frac{\left(4 d^{3} x^{3} b^{3}+6 \mathrm{I} b^{2} d^{3} x^{2}+12 b^{3} c d^{2} x^{2}+12 \mathrm{I} b^{2} c d^{2} x+12 b^{3} c^{2} d x+6 \mathrm{I} c^{2} d b^{2}+4 b^{3} c^{3}-6 b d^{3} x-3 \mathrm{I} d^{3}-6 c d^{2} b\right) \mathrm{e}^{2 \mathrm{I}(b x+a)}}{8 b^{4}}$

$$
\begin{aligned}
& -\frac{\left(4 d^{3} x^{3} b^{3}-6 \mathrm{I} b^{2} d^{3} x^{2}+12 b^{3} c d^{2} x^{2}-12 \mathrm{I} b^{2} c d^{2} x+12 b^{3} c^{2} d x-6 \mathrm{I} c^{2} d b^{2}+4 b^{3} c^{3}-6 b d^{3} x+3 \mathrm{I} d^{3}-6 c d^{2} b\right) \mathrm{e}^{-2 \mathrm{I}(b x+a)}}{8 b^{4}}+\mathrm{I} c^{3} x \\
& +\frac{6 a c^{2} d \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{6 a^{2} c d^{2} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{\mathrm{I} d^{3} x^{4}}{4}-\frac{2 c^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b}-\mathrm{I} c d^{2} x^{3}-\frac{3 \mathrm{I} c^{2} d x^{2}}{2}-\frac{3 \mathrm{I} c d^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{b^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{d^{3} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{3}}{b}+\frac{2 a^{3} d^{3} \ln \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{4}}-\frac{3 \mathrm{I} a^{4} d^{3}}{2 b^{4}}-\frac{3 \mathrm{I} c^{2} d a^{2}}{b^{2}}+\frac{4 \mathrm{I} c d^{2} a^{3}}{b^{3}}-\frac{2 \mathrm{I} d^{3} a^{3} x}{b^{3}}+\frac{6 \mathrm{I} c d^{2} a^{2} x}{b^{2}}-\frac{6 \mathrm{I} c^{2} d a x}{b} \\
& +\frac{3 d^{3} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x}{2 b^{3}}+\frac{3 c d^{2} \operatorname{poly} \log \left(3,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{3}}+\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(4,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b^{4}}+\frac{c^{3} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}{b} \\
& +\frac{3 c d^{2} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x^{2}}{b}+\frac{3 c^{2} d \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right) x}{b}-\frac{3 \mathrm{I} c^{2} d \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 b^{2}}-\frac{3 \mathrm{I} d^{3} \operatorname{poly} \log \left(2,-\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) x^{2}}{2 b^{2}}
\end{aligned}
$$

Problem 106: Result more than twice size of optimal antiderivative.

$$
\int(d x+c)^{2} \sec (b x+a)^{2} \sin (3 b x+3 a) \mathrm{d} x
$$

Optimal(type 4, 136 leaves, 15 steps):
$-\frac{4 \mathrm{I} d(d x+c) \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}+\frac{8 d^{2} \cos (b x+a)}{b^{3}}-\frac{4(d x+c)^{2} \cos (b x+a)}{b}+\frac{2 \mathrm{I} d^{2} \operatorname{poly} \log \left(2,-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 \mathrm{I} d^{2} \operatorname{polylog}\left(2, \mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}$

$$
-\frac{(d x+c)^{2} \sec (b x+a)}{b}+\frac{8 d(d x+c) \sin (b x+a)}{b^{2}}
$$

Result(type 4, 344 leaves):

$$
\begin{aligned}
&-\frac{2\left(d^{2} x^{2} b^{2}+2 b^{2} c d x+b^{2} c^{2}+2 \mathrm{I} b d^{2} x-2 d^{2}+2 \mathrm{I} b c d\right) \mathrm{e}^{\mathrm{I}(b x+a)}}{b^{3}}-\frac{2\left(d^{2} x^{2} b^{2}+2 b^{2} c d x+b^{2} c^{2}-2 \mathrm{I} b d^{2} x-2 d^{2}-2 \mathrm{I} b c d\right) \mathrm{e}^{-\mathrm{I}(b x+a)}}{b^{3}} \\
&-\frac{2\left(x^{2} d^{2}+2 c d x+c^{2}\right) \mathrm{e}^{\mathrm{I}(b x+a)}}{b\left(\mathrm{e}^{2 \mathrm{I}(b x+a)}+1\right)}-\frac{4 \mathrm{I} d c \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{2}}-\frac{2 d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) x}{b^{2}}-\frac{2 d^{2} \ln \left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}+\frac{2 d^{2} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) x}{b^{2}} \\
&+\frac{2 d^{2} \ln \left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right) a}{b^{3}}+\frac{2 \mathrm{I} d^{2} \operatorname{dilog}\left(1+\mathrm{Ie}^{\mathrm{I}(b x+a)}\right)}{b^{3}}-\frac{2 \mathrm{I} d^{2} \operatorname{dilog}\left(1-\mathrm{I} \mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}+\frac{4 \mathrm{I} d^{2} a \arctan \left(\mathrm{e}^{\mathrm{I}(b x+a)}\right)}{b^{3}}
\end{aligned}
$$

Test results for the 3 problems in "4.7.4 x^m (a+b trig^n)^p.txt"
Problem 1: Result is not expressed in closed-form.

$$
\int \frac{x}{a+b \sin (x)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 153 leaves, 9 steps):

$$
\begin{aligned}
& -\frac{\mathrm{I} x \ln \left(1-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}+\frac{\mathrm{I} x \ln \left(1-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}-\frac{\operatorname{poly} \log \left(2, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}} \\
& \quad+\frac{\operatorname{poly} \log \left(2, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}}
\end{aligned}
$$

Result(type 7, 71 leaves):

$$
-\left(\sum_{-R I=\operatorname{RootOf}\left(b \_Z^{4}+(-4 a-2 b) \_Z^{2}+b\right)} \frac{\mathrm{I} x \ln \left(\frac{R 1-\mathrm{e}^{\mathrm{I} x}}{R 1}\right)+\operatorname{dilog}\left(\frac{R 1-\mathrm{e}^{\mathrm{I} x}}{R 1}\right)}{-\_^{2} b+2 a+b}\right)
$$

Problem 2: Unable to integrate problem.

$$
\int \frac{x^{3}}{a+b \sin (x)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 311 leaves, 13 steps):

$$
\begin{aligned}
& -\frac{\mathrm{I} x^{3} \ln \left(1-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}+\frac{\mathrm{I} x^{3} \ln \left(1-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}-\frac{3 x^{2} \operatorname{polylog}\left(2, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}} \\
& +\frac{3 x^{2} \operatorname{polylog}\left(2, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}}-\frac{3 \mathrm{I} x \operatorname{poly} \log \left(3, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}}+\frac{3 \mathrm{I} x \operatorname{poly} \log \left(3, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}} \\
& +\frac{3 \operatorname{polylog}\left(4, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{8 \sqrt{a} \sqrt{a+b}}-\frac{3 \operatorname{polylog}\left(4, \frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{8 \sqrt{a} \sqrt{a+b}}
\end{aligned}
$$

Result(type 8, 16 leaves):

$$
\int \frac{x^{3}}{a+b \sin (x)^{2}} \mathrm{~d} x
$$

Problem 3: Unable to integrate problem.

$$
\int \frac{x^{2}}{a+b \cos (x)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 235 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{\mathrm{I} x^{2} \ln \left(1+\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}+\frac{\mathrm{I} x^{2} \ln \left(1+\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}-\frac{x \operatorname{polylog}\left(2,-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}} \\
& +\frac{x \operatorname{polylog}\left(2,-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{2 \sqrt{a} \sqrt{a+b}}-\frac{\mathrm{I} \operatorname{poly} \log \left(3,-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}}+\frac{\mathrm{Ipolylog}\left(3,-\frac{b \mathrm{e}^{2 \mathrm{I} x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right)}{4 \sqrt{a} \sqrt{a+b}}
\end{aligned}
$$

Result(type 8, 16 leaves):

$$
\int \frac{x^{2}}{a+b \cos (x)^{2}} \mathrm{~d} x
$$

Test results for the 86 problems in "4.7.5 $x^{\wedge} m$ trig(a+b log(c $\left.\left.x^{\wedge} n\right)\right)^{\wedge} p . t x t^{\prime \prime}$
Problem 1: Unable to integrate problem.

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal (type 3, 57 leaves, 1 step):

$$
-\frac{b n x^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right)}{b^{2} n^{2}+9}+\frac{3 x^{3} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{b^{2} n^{2}+9}
$$

Result (type 8, 17 leaves):

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right) d x
$$

Problem 2: Unable to integrate problem.

$$
\int \sin \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal(type 3, 52 leaves, 1 step):

$$
-\frac{b n x \cos \left(a+b \ln \left(c x^{n}\right)\right)}{b^{2} n^{2}+1}+\frac{x \sin \left(a+b \ln \left(c x^{n}\right)\right)}{b^{2} n^{2}+1}
$$

Result (type 8, 13 leaves):

$$
\int \sin \left(a+b \ln \left(c x^{n}\right)\right) d x
$$

Problem 3: Unable to integrate problem.

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 95 leaves, 2 steps):

$$
\frac{2 b^{2} n^{2} x^{3}}{3\left(4 b^{2} n^{2}+9\right)}-\frac{2 b n x^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)}{4 b^{2} n^{2}+9}+\frac{3 x^{3} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{4 b^{2} n^{2}+9}
$$

Result(type 8, 19 leaves):

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2} d x
$$

Problem 5: Unable to integrate problem.

$$
\int \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 149 leaves, 2 steps):

$$
-\frac{6 b^{3} n^{3} x \cos \left(a+b \ln \left(c x^{n}\right)\right)}{9 b^{4} n^{4}+10 b^{2} n^{2}+1}+\frac{6 b^{2} n^{2} x \sin \left(a+b \ln \left(c x^{n}\right)\right)}{9 b^{4} n^{4}+10 b^{2} n^{2}+1}-\frac{3 b n x \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{9 b^{2} n^{2}+1}+\frac{x \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{9 b^{2} n^{2}+1}
$$

Result(type 8, 15 leaves):

$$
\int \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3} \mathrm{~d} x
$$

Problem 7: Unable to integrate problem.

$$
\int \frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 158 leaves, 2 steps):

$$
-\frac{6 b^{3} n^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{4} n^{4}+10 b^{2} n^{2}+1\right) x}-\frac{6 b^{2} n^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{4} n^{4}+10 b^{2} n^{2}+1\right) x}-\frac{3 b n \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{\left(9 b^{2} n^{2}+1\right) x}-\frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{\left(9 b^{2} n^{2}+1\right) x}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Problem 8: Unable to integrate problem.

$$
\int \frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 158 leaves, 2 steps):

$$
-\frac{6 b^{3} n^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{4} n^{4}+40 b^{2} n^{2}+16\right) x^{2}}-\frac{12 b^{2} n^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{4} n^{4}+40 b^{2} n^{2}+16\right) x^{2}}-\frac{3 b n \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{\left(9 b^{2} n^{2}+4\right) x^{2}}-\frac{2 \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{\left(9 b^{2} n^{2}+4\right) x^{2}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{x^{3}} \mathrm{~d} x
$$

Problem 9: Unable to integrate problem.

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{4} \mathrm{~d} x
$$

Optimal(type 3, 202 leaves, 3 steps):
$\frac{8 b^{4} n^{4} x^{3}}{64 b^{4} n^{4}+180 b^{2} n^{2}+81}-\frac{24 b^{3} n^{3} x^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)}{64 b^{4} n^{4}+180 b^{2} n^{2}+81}+\frac{36 b^{2} n^{2} x^{3} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{64 b^{4} n^{4}+180 b^{2} n^{2}+81}$

$$
-\frac{4 b n x^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{16 b^{2} n^{2}+9}+\frac{3 x^{3} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{4}}{16 b^{2} n^{2}+9}
$$

Result(type 8, 19 leaves):

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{4} \mathrm{~d} x
$$

Problem 10: Unable to integrate problem.

$$
\int \frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{4}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 202 leaves, 3 steps):

$$
\begin{aligned}
& -\frac{24 b^{4} n^{4}}{\left(64 b^{4} n^{4}+20 b^{2} n^{2}+1\right) x}-\frac{24 b^{3} n^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)}{\left(64 b^{4} n^{4}+20 b^{2} n^{2}+1\right) x}-\frac{12 b^{2} n^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{\left(64 b^{4} n^{4}+20 b^{2} n^{2}+1\right) x} \\
& -\frac{4 b n \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{\left(16 b^{2} n^{2}+1\right) x}-\frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{4}}{\left(16 b^{2} n^{2}+1\right) x}
\end{aligned}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{4}}{x^{2}} \mathrm{~d} x
$$

Problem 11: Unable to integrate problem.

$$
\int \sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right) \mathrm{d} x
$$

Optimal(type 3, 69 leaves, 3 steps):


Result(type 8, 19 leaves):

$$
\int \sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right) \mathrm{d} x
$$

Problem 12: Unable to integrate problem.

$$
\int x \sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 68 leaves, 3 steps):

$$
\frac{x^{2}}{4}-\frac{x^{2}\left(c x^{n}\right)^{\frac{2}{n}}}{16 \mathrm{e}^{2 a n} \sqrt{-\frac{1}{n^{2}}}}-\frac{\mathrm{e}^{2 a n \sqrt{-\frac{1}{n^{2}}} x^{2} \ln (x)}}{4\left(c x^{n}\right)^{\frac{2}{n}}}
$$

Result(type 8, 23 leaves):

$$
\int x \sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right)^{2} \mathrm{~d} x
$$

Problem 13: Unable to integrate problem.

$$
\int \frac{\sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right)^{2}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 68 leaves, 3 steps):

$$
-\frac{1}{4 x^{2}}+\frac{\mathrm{e}^{2 a n \sqrt{-\frac{1}{n^{2}}}}}{16 x^{2}\left(c x^{n}\right)^{\frac{2}{n}}}-\frac{\left(c x^{n}\right)^{\frac{2}{n}} \ln (x)}{2 a n \sqrt{-\frac{1}{n^{2}}} x^{2}}
$$

Result(type 8, 25 leaves):

$$
\int \frac{\sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right)^{2}}{x^{3}} \mathrm{~d} x
$$

Problem 14: Unable to integrate problem.

$$
\int x^{2} \sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 149 leaves, 3 steps):

$$
-\frac{3 \mathrm{e}^{a n \sqrt{-\frac{1}{n^{2}}}} n x^{3} \sqrt{-\frac{1}{n^{2}}}}{16\left(c x^{n}\right)^{\frac{1}{n}}}+\frac{3 n x^{3}\left(c x^{n}\right)^{\frac{1}{n}} \sqrt{-\frac{1}{n^{2}}}}{32 \mathrm{e}^{a n \sqrt{-\frac{1}{n^{2}}}}}-\frac{n x^{3}\left(c x^{n}\right)^{\frac{3}{n}} \sqrt{-\frac{1}{n^{2}}}}{48 \mathrm{e}^{3 a n \sqrt{-\frac{1}{n^{2}}}}}+\frac{\mathrm{e}^{3 a n \sqrt{-\frac{1}{n^{2}}} n x^{3} \ln (x) \sqrt{-\frac{1}{n^{2}}}}}{8\left(c x^{n}\right)^{\frac{3}{n}}}
$$

Result(type 8, 25 leaves):

$$
\int x^{2} \sin \left(a+\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}\right)^{3} \mathrm{~d} x
$$

Problem 15: Unable to integrate problem.

$$
\int \frac{\sin \left(a+\frac{\ln \left(c x^{n}\right) \sqrt{-\frac{1}{n^{2}}}}{3}\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 149 leaves, 3 steps):

$$
-\frac{\mathrm{e}^{3 a n \sqrt{-\frac{1}{n^{2}}}} \sqrt[n]{-\frac{1}{n^{2}}}}{16 x\left(c x^{n}\right)^{\frac{1}{n}}}+\frac{9 \mathrm{e}^{a n \sqrt{-\frac{1}{n^{2}}}} n \sqrt{-\frac{1}{n^{2}}}}{32 x\left(c x^{n}\right)^{\frac{1}{3 n}}}-\frac{9 n\left(c x^{n}\right)^{\frac{1}{3 n}} \sqrt{-\frac{1}{n^{2}}}}{16 \mathrm{e}^{a n \sqrt{-\frac{1}{n^{2}}}} x}-\frac{n\left(c x^{n}\right)^{\frac{1}{n}} \ln (x) \sqrt{-\frac{1}{n^{2}}}}{8}
$$

Result(type 8, 26 leaves):


Problem 16: Unable to integrate problem.

$$
\int x^{m} \sin \left(a+\frac{\ln \left(c x^{2}\right) \sqrt{-(1+m)^{2}}}{2}\right) \mathrm{d} x
$$

Optimal(type 3, 92 leaves, 3 steps):

$$
-\frac{\mathrm{e}^{\frac{a(1+m)}{\sqrt{-(1+m)^{2}}}} x^{1+m}\left(c x^{2}\right)^{\frac{1}{2}+\frac{m}{2}}}{4 \sqrt{-(1+m)^{2}}}+\frac{\mathrm{e}^{\frac{a \sqrt{-(1+m)^{2}}}{1+m}}(1+m) x^{1+m}\left(c x^{2}\right)^{-\frac{1}{2}-\frac{m}{2}} \ln (x)}{2 \sqrt{-(1+m)^{2}}}
$$

Result(type 8, 26 leaves):

$$
\int x^{m} \sin \left(a+\frac{\ln \left(c x^{2}\right) \sqrt{-(1+m)^{2}}}{2}\right) \mathrm{d} x
$$

Problem 17: Unable to integrate problem.

$$
\int x^{m} \sin \left(a+\frac{\ln \left(c x^{2}\right) \sqrt{-(1+m)^{2}}}{4}\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 90 leaves, 3 steps):

$$
\frac{x^{1+m}}{2(1+m)}-\frac{\mathrm{e}^{\frac{2 a(1+m)}{\sqrt{-(1+m)^{2}}} x^{1+m}\left(c x^{2}\right)^{\frac{1}{2}+\frac{m}{2}}}}{8(1+m)}-\frac{x^{1+m}\left(c x^{2}\right)^{-\frac{1}{2}-\frac{m}{2}} \ln (x)}{4 \mathrm{e}^{\sqrt{-(1+m)^{2}}}}
$$

Result(type 8, 28 leaves):

$$
\int x^{m} \sin \left(a+\frac{\ln \left(c x^{2}\right) \sqrt{-(1+m)^{2}}}{4}\right)^{2} \mathrm{~d} x
$$

Problem 18: Unable to integrate problem.

$$
\int \frac{\sqrt{\sin \left(a+b \ln \left(c x^{n}\right)\right)}}{x^{2}} d x
$$

Optimal(type 5, 90 leaves, 3 steps):

$$
-\frac{2 \text { hypergeom }\left(\left[-\frac{1}{2},-\frac{1}{4}+\frac{\mathrm{I}}{2 b n}\right],\left[\frac{3}{4}+\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right) \sqrt{\sin \left(a+b \ln \left(c x^{n}\right)\right)}}{(2+\mathrm{I} b n) x \sqrt{1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\sqrt{\sin \left(a+b \ln \left(c x^{n}\right)\right)}}{x^{2}} \mathrm{~d} x
$$

Problem 19: Unable to integrate problem.

$$
\int \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
\frac{2 x \text { hypergeom }\left(\left[-\frac{3}{2},-\frac{3}{4}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{1}{4}-\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}{(2-3 \mathrm{I} b n)\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{3 / 2}}
$$

Result(type 8, 15 leaves):

$$
\int \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2} \mathrm{~d} x
$$

Problem 21: Unable to integrate problem.

$$
\int \frac{1}{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
\frac{2 x\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{3 / 2} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{3}{4}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{7}{4}-\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2+3 \mathrm{I} b n) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{\sin \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 23: Unable to integrate problem.

$$
\int(e x)^{m} \sin \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{4} \mathrm{~d} x
$$

Optimal(type 3, 337 leaves, 3 steps):

$$
\begin{aligned}
& \frac{24 b^{4} d^{4} n^{4}(e x)^{1+m}}{e(1+m)\left((1+m)^{2}+4 b^{2} d^{2} n^{2}\right)\left((1+m)^{2}+16 b^{2} d^{2} n^{2}\right)}-\frac{24 b^{3} d^{3} n^{3}(e x)^{1+m} \cos \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right) \sin \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)}{e\left((1+m)^{2}+4 b^{2} d^{2} n^{2}\right)\left((1+m)^{2}+16 b^{2} d^{2} n^{2}\right)} \\
& +\frac{12 b^{2} d^{2}(1+m) n^{2}(e x)^{1+m} \sin \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{2}}{e\left((1+m)^{2}+4 b^{2} d^{2} n^{2}\right)\left((1+m)^{2}+16 b^{2} d^{2} n^{2}\right)}-\frac{4 b d n(e x)^{1+m} \cos \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right) \sin \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{3}}{e\left((1+m)^{2}+16 b^{2} d^{2} n^{2}\right)} \\
& +\frac{(1+m)(e x)^{1+m} \sin \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{4}}{e\left((1+m)^{2}+16 b^{2} d^{2} n^{2}\right)}
\end{aligned}
$$

Result(type 8, 23 leaves):

$$
\int(e x)^{m} \sin \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{4} \mathrm{~d} x
$$

Problem 24: Unable to integrate problem.

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 102 leaves, 3 steps):

$$
\frac{x^{3} \text { hypergeom }\left(\left[-p, \frac{-3 \mathrm{I}-b n p}{2 b n}\right],\left[1-\frac{3 \mathrm{I}}{2 b n}-\frac{p}{2}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{p}}{(3-\mathrm{I} b n p)\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{p}}
$$

Result(type 8, 19 leaves):

$$
\int x^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Problem 25: Unable to integrate problem.

$$
\int x \sin \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 99 leaves, 3 steps):

$$
\frac{x^{2} \text { hypergeom }\left(\left[-p,-\frac{\mathrm{I}}{b n}-\frac{p}{2}\right],\left[1-\frac{\mathrm{I}}{b n}-\frac{p}{2}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)^{p}}{(2-\mathrm{I} b n p)\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{p}}
$$

Result (type 8, 17 leaves):

$$
\int x \sin \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Problem 26: Unable to integrate problem.

$$
\int x^{2} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 95 leaves, 2 steps):

$$
\frac{2 b^{2} n^{2} x^{3}}{3\left(4 b^{2} n^{2}+9\right)}+\frac{3 x^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{4 b^{2} n^{2}+9}+\frac{2 b n x^{3} \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)}{4 b^{2} n^{2}+9}
$$

Result(type 8, 19 leaves):

$$
\int x^{2} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2} d x
$$

Problem 27: Unable to integrate problem.

$$
\int x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 92 leaves, 2 steps):

$$
\frac{b^{2} n^{2} x^{2}}{4\left(b^{2} n^{2}+1\right)}+\frac{x^{2} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{2\left(b^{2} n^{2}+1\right)}+\frac{b n x^{2} \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)}{2\left(b^{2} n^{2}+1\right)}
$$

Result (type 8, 17 leaves):

$$
\int x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2} \mathrm{~d} x
$$

Problem 28: Unable to integrate problem.

$$
\int \frac{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 158 leaves, 2 steps):

$$
-\frac{6 b^{2} n^{2} \cos \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{4} n^{4}+10 b^{2} n^{2}+1\right) x}-\frac{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{\left(9 b^{2} n^{2}+1\right) x}+\frac{6 b^{3} n^{3} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{4} n^{4}+10 b^{2} n^{2}+1\right) x}+\frac{3 b n \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{\left(9 b^{2} n^{2}+1\right) x}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Problem 29: Unable to integrate problem.

$$
\int \cos \left(a+b \ln \left(c x^{n}\right)\right)^{4} \mathrm{~d} x
$$

Optimal(type 3, 191 leaves, 3 steps):

$$
\begin{aligned}
& \frac{24 b^{4} n^{4} x}{64 b^{4} n^{4}+20 b^{2} n^{2}+1}+\frac{12 b^{2} n^{2} x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{64 b^{4} n^{4}+20 b^{2} n^{2}+1}+\frac{x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{4}}{16 b^{2} n^{2}+1}+\frac{24 b^{3} n^{3} x \cos \left(a+b \ln \left(c x^{n}\right)\right) \sin \left(a+b \ln \left(c x^{n}\right)\right)}{64 b^{4} n^{4}+20 b^{2} n^{2}+1} \\
& +\frac{4 b n x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{3} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{16 b^{2} n^{2}+1}
\end{aligned}
$$

Result(type 8, 15 leaves):

$$
\int \cos \left(a+b \ln \left(c x^{n}\right)\right)^{4} \mathrm{~d} x
$$

Problem 31: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}{x} \mathrm{~d} x
$$

Optimal(type 4, 93 leaves, 3 steps):

$$
\frac{2 \sqrt{\cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right), \sqrt{2}\right)}{3 \cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right) b n}+\frac{2 \sin \left(a+b \ln \left(c x^{n}\right)\right) \sqrt{\cos \left(a+b \ln \left(c x^{n}\right)\right)}}{3 b n}
$$

Result(type 4, 246 leaves):

$$
\begin{aligned}
& -\left(2 \sqrt { ( 2 \operatorname { c o s } ( \frac { a } { 2 } + \frac { b \operatorname { l n } ( c x ^ { n } ) } { 2 } ) ^ { 2 } - 1 ) \operatorname { s i n } ( \frac { a } { 2 } + \frac { b \operatorname { l n } ( c x ^ { n } ) } { 2 } ) ^ { 2 } } \left(4 \cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right) \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{4}\right.\right. \\
& \quad+\sqrt{\sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}} \sqrt{2 \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}-1} \operatorname{EllipticF}\left(\cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right), \sqrt{2}\right)-2 \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2} \cos \left(\frac{a}{2}\right. \\
& \left.\left.\left.\quad+\frac{b \ln \left(c x^{n}\right)}{2}\right)\right)\right) /\left(3 n \sqrt{-2 \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{4}+\sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}} \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right) \sqrt{\left.2 \cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}-1 b\right)}\right.
\end{aligned}
$$

Problem 32: Result unnecessarily involves higher level functions.

$$
\int \frac{1}{x \sqrt{\cos \left(a+b \ln \left(c x^{n}\right)\right)}} \mathrm{d} x
$$

Optimal(type 4, 60 leaves, 2 steps):

$$
\frac{2 \sqrt{\cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right), \sqrt{2}\right)}{\cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right) b n}
$$

Result(type 5, 25 leaves):

$$
\frac{2 \text { InverseJacobiAM }\left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}, \sqrt{2}\right)}{b n}
$$

Problem 33: Unable to integrate problem.

$$
\int \frac{1}{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
\frac{2 x\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{3 / 2} \text { hypergeom }\left(\left[\frac{3}{2}, \frac{3}{4}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{7}{4}-\frac{\mathrm{I}}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2+3 \mathrm{I} b n) \cos \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 35: Unable to integrate problem.

$$
\int \frac{1}{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
\frac{2 x\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{5 / 2} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{5}{4}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{9}{4}-\frac{\mathrm{I}}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2+5 \mathrm{I} b n) \cos \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}} \mathrm{~d} x
$$

Problem 36: Unable to integrate problem.

$$
\int \frac{1}{\cos (a-2 \mathrm{I} \ln (c x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 42 leaves, 3 steps):

$$
\frac{-1-c^{4} \mathrm{e}^{2 \mathrm{I} a} x^{4}}{2 c^{4} \mathrm{e}^{2 \mathrm{I} a} x^{3} \cos (a-2 \mathrm{I} \ln (c x))^{3 / 2}}
$$

Result(type 8, 14 leaves):

$$
\int \frac{1}{\cos (a-2 \operatorname{In}(c x))^{3 / 2}} \mathrm{~d} x
$$

Problem 37: Unable to integrate problem.

$$
\int x^{m} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 201 leaves, 2 steps):
$\frac{6 b^{2}(1+m) n^{2} x^{1+m} \cos \left(a+b \ln \left(c x^{n}\right)\right)}{\left((1+m)^{2}+b^{2} n^{2}\right)\left((1+m)^{2}+9 b^{2} n^{2}\right)}+\frac{(1+m) x^{1+m} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{3}}{(1+m)^{2}+9 b^{2} n^{2}}+\frac{6 b^{3} n^{3} x^{1+m} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{\left((1+m)^{2}+b^{2} n^{2}\right)\left((1+m)^{2}+9 b^{2} n^{2}\right)}$
$+\frac{3 b n x^{1+m} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{2} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{(1+m)^{2}+9 b^{2} n^{2}}$
Result(type 8, 19 leaves):

$$
\int x^{m} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{3} \mathrm{~d} x
$$

Problem 38: Unable to integrate problem.

$$
\int x^{m} \cos \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal (type 3, 70 leaves, 1 step):

$$
\frac{(1+m) x^{1+m} \cos \left(a+b \ln \left(c x^{n}\right)\right)}{(1+m)^{2}+b^{2} n^{2}}+\frac{b n x^{1+m} \sin \left(a+b \ln \left(c x^{n}\right)\right)}{(1+m)^{2}+b^{2} n^{2}}
$$

Result(type 8, 17 leaves):

$$
\int x^{m} \cos \left(a+b \ln \left(c x^{n}\right)\right) d x
$$

Problem 39: Unable to integrate problem.

$$
\int \frac{x^{m}}{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 111 leaves, 3 steps):

$$
\frac{2 x^{1+m}\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{5 / 2} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{-2 \mathrm{I}-2 \mathrm{I} m+5 b n}{4 b n}\right],\left[\frac{-2 \mathrm{I}-2 \mathrm{I} m+9 b n}{4 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2+2 m+5 \mathrm{I} b n) \cos \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{x^{m}}{\cos \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}} \mathrm{~d} x
$$

Problem 40: Unable to integrate problem.

$$
\int x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 99 leaves, 3 steps):

$$
\frac{x^{2} \cos \left(a+b \ln \left(c x^{n}\right)\right)^{p} \text { hypergeom }\left(\left[-p,-\frac{\mathrm{I}}{b n}-\frac{p}{2}\right],\left[1-\frac{\mathrm{I}}{b n}-\frac{p}{2}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2-\mathrm{I} b n p)\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{p}}
$$

Result(type 8, 17 leaves):

$$
\int x \cos \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Problem 41: Unable to integrate problem.

$$
\int \frac{\tan (a+\mathrm{I} \ln (x))}{x^{4}} \mathrm{~d} x
$$

Optimal(type 3, 40 leaves, 5 steps):

$$
\frac{\mathrm{I}}{3 x^{3}}-\frac{2 \mathrm{I}}{\mathrm{e}^{2 \mathrm{I} a} x}-\frac{2 \mathrm{I} \arctan \left(\frac{x}{\mathrm{e}^{\mathrm{I} a}}\right)}{\mathrm{e}^{3 \mathrm{I} a}}
$$

Result (type 8, 34 leaves):

$$
\frac{\mathrm{I}}{3 x^{3}}-\mathrm{I}\left(\int-\frac{2}{x^{4}\left(\left(\mathrm{e}^{\mathrm{I}(a+\mathrm{Iln}(x))}\right)^{2}+1\right)} \mathrm{d} x\right)
$$

Problem 43: Unable to integrate problem.

$$
\int \frac{\tan (a+\mathrm{I} \ln (x))^{2}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 54 leaves, 5 steps):

$$
\frac{\mathrm{e}^{2 \mathrm{I} a}}{x\left(\mathrm{e}^{2 \mathrm{I} a}+x^{2}\right)}+\frac{3 x}{\mathrm{e}^{2 \mathrm{I} a}+x^{2}}+\frac{2 \arctan \left(\frac{x}{\mathrm{e}^{\mathrm{I} a}}\right)}{\mathrm{e}^{\mathrm{I} a}}
$$

Result(type 8, 52 leaves):

$$
\frac{1}{x}+\frac{2}{\left(\left(\mathrm{e}^{\mathrm{I}(a+\mathrm{I} \ln (x))}\right)^{2}+1\right) x}-\left(\int-\frac{2}{\left(\left(\mathrm{e}^{\mathrm{I}(a+\mathrm{I} \ln (x))}\right)^{2}+1\right) x^{2}} \mathrm{~d} x\right)
$$

Problem 44: Unable to integrate problem.

$$
\int(e x)^{m} \tan (a+\mathrm{I} \ln (x))^{3} \mathrm{~d} x
$$

Optimal(type 5, 156 leaves, 6 steps):

$$
\begin{gathered}
-\frac{\mathrm{I}(1-m) m x(e x)^{m}}{2(1+m)}+\frac{\mathrm{I}\left(1-\frac{\mathrm{e}^{2 \mathrm{I} a}}{x^{2}}\right)^{2} x(e x)^{m}}{2\left(1+\frac{\mathrm{e}^{2 \mathrm{I} a}}{x^{2}}\right)^{2}}+\frac{\mathrm{I}\left(\mathrm{e}^{2 \mathrm{I} a}(3+m)+\frac{\mathrm{e}^{4 \mathrm{I} a}(1-m)}{x^{2}}\right) x(e x)^{m}}{2 \mathrm{e}^{2 \mathrm{I} a}\left(1+\frac{\mathrm{e}^{2 \mathrm{I} a}}{x^{2}}\right)} \\
-\frac{\mathrm{I}\left(m^{2}+2 m+3\right) x(e x)^{m} \text { hypergeom }\left(\left[1,-\frac{1}{2}-\frac{m}{2}\right],\left[\frac{1}{2}-\frac{m}{2}\right],-\frac{\mathrm{e}^{2 \mathrm{I} a}}{x^{2}}\right)}{1+m}
\end{gathered}
$$

Result(type 8, 18 leaves):

$$
\int(e x)^{m} \tan (a+\mathrm{I} \ln (x))^{3} \mathrm{~d} x
$$

Problem 45: Unable to integrate problem.

$$
\int \tan (a+\ln (x))^{p} d x
$$

Optimal(type 6, 96 leaves, 4 steps):

$$
\frac{\left(\frac{\mathrm{I}\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I}}\right)}{1+\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I}}}\right)^{p}\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I}}\right)^{p} x \operatorname{AppellF} 1\left(-\frac{\mathrm{I}}{2},-p, p, 1-\frac{\mathrm{I}}{2}, \mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I}},-\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I}}\right)}{\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I}}\right)^{p}}
$$

Result(type 8, 9 leaves):

$$
\int \tan (a+\ln (x))^{p} \mathrm{~d} x
$$

Problem 46: Unable to integrate problem.

$$
\int \tan (a+2 \ln (x))^{p} \mathrm{~d} x
$$

Optimal(type 6, 96 leaves, 4 steps):

$$
\frac{\left(\frac{\mathrm{I}\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)}{1+\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}}\right)^{p}\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)^{p} x \operatorname{AppellF1}\left(-\frac{\mathrm{I}}{4},-p, p, 1-\frac{\mathrm{I}}{4}, \mathrm{e}^{\mathrm{I} a} x^{4 \mathrm{I}},-\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)}{\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)^{p}}
$$

Result(type 8, 11 leaves):

$$
\int \tan (a+2 \ln (x))^{p} \mathrm{~d} x
$$

Problem 47: Unable to integrate problem.

$$
\int x^{3} \tan \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 145 leaves, 5 steps):

$$
\frac{(4 \mathrm{I}-b d n) x^{4}}{4 b d n}+\frac{\mathrm{I} x^{4}\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}-\frac{2 \mathrm{I} x^{4} \text { hypergeom }\left(\left[1, \frac{-2 \mathrm{I}}{b d n}\right],\left[1-\frac{2 \mathrm{I}}{b d n}\right],-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n}
$$

Result(type 8, 196 leaves):

$$
\begin{aligned}
-\frac{x^{4}}{4} & +\frac{2 \mathrm{I} x^{4}}{d b n\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x))}\right)\right.}{2}\right)\right)\right)^{2}}+1\right.\right.}+1 \\
& \left.\int \frac{8 \mathrm{I} x^{3}}{d b n\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2}}+1\right)\right.} \mathrm{dx}\right)
\end{aligned}
$$

Problem 48: Unable to integrate problem.

$$
\int x \tan \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 145 leaves, 5 steps):

$$
\frac{(2 \mathrm{I}-b d n) x^{2}}{2 b d n}+\frac{\mathrm{I} x^{2}\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}-\frac{2 \mathrm{I} x^{2} \text { hypergeom }\left(\left[1, \frac{-\mathrm{I}}{b d n}\right],\left[1-\frac{\mathrm{I}}{b d n}\right],-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n}
$$

Result(type 8, 194 leaves):

$$
\begin{aligned}
& -\frac{x^{2}}{2}+\frac{2 \mathrm{I} x^{2}}{d b n\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2}}+1\right)\right.}-( \\
& \left.\left.\int \frac{4 \mathrm{I} x}{\left.\left.d b n\left(\left(\mathrm{e}^{\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right.\right.\right.}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2}}+1\right) \mathrm{dx}\right)
\end{aligned}
$$

Problem 53: Unable to integrate problem.

$$
\int(e x)^{m} \cot (a+b \ln (x))^{p} \mathrm{~d} x
$$

Optimal(type 6, 137 leaves, 4 steps):

$$
\frac{(e x)^{1+m}\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I} b}\right)^{p}\left(\frac{-\mathrm{I}\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I} b}\right)}{1-\mathrm{e}^{2 \mathrm{I} a} x^{\mathrm{I} b}}\right)^{p} \text { AppellF1 }\left(\frac{-\frac{\mathrm{I}}{2}(1+m)}{b}, p,-p, 1-\frac{\mathrm{I}(1+m)}{2 b}, \mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I} b},-\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I} b}\right)}{e(1+m)\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{2 \mathrm{I} b}\right)^{p}}
$$

Result(type 8, 17 leaves):

$$
\int(e x)^{m} \cot (a+b \ln (x))^{p} \mathrm{~d} x
$$

Problem 54: Unable to integrate problem.

$$
\int \cot (a+2 \ln (x))^{p} \mathrm{~d} x
$$

Optimal(type 6, 96 leaves, 4 steps):

$$
\frac{\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)^{p}\left(\frac{-\mathrm{I}\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)}{1-\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}}\right)^{p} x \operatorname{AppellF} 1\left(-\frac{\mathrm{I}}{4}, p,-p, 1-\frac{\mathrm{I}}{4}, \mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}},-\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)}{\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{4 \mathrm{I}}\right)^{p}}
$$

Result(type 8, 11 leaves):

$$
\int \cot (a+2 \ln (x))^{p} \mathrm{~d} x
$$

Problem 55: Unable to integrate problem.

$$
\int \cot (a+3 \ln (x))^{p} \mathrm{~d} x
$$

Optimal(type 6, 96 leaves, 4 steps):

$$
\frac{\left(1-\mathrm{e}^{2 \mathrm{I} a} x^{6 \mathrm{I}}\right)^{p}\left(\frac{-\mathrm{I}\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{6 \mathrm{I}}\right)}{1-\mathrm{e}^{2 \mathrm{I} a} x^{6 \mathrm{I}}}\right)^{p} x \operatorname{AppellF} 1\left(-\frac{\mathrm{I}}{6}, p,-p, 1-\frac{\mathrm{I}}{6}, \mathrm{e}^{2 \mathrm{I} a} x^{6 \mathrm{I}},-\mathrm{e}^{2 \mathrm{I} a} x^{6 \mathrm{I}}\right)}{\left(1+\mathrm{e}^{2 \mathrm{I} a} x^{6 \mathrm{I}}\right)^{p}}
$$

Result(type 8, 11 leaves):

$$
\int \cot (a+3 \ln (x))^{p} \mathrm{~d} x
$$

Problem 56: Unable to integrate problem.

$$
\int x^{2} \cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 144 leaves, 5 steps):

$$
\frac{(3 \mathrm{I}-b d n) x^{3}}{3 b d n}+\frac{\mathrm{I} x^{3}\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}-\frac{2 \mathrm{I} x^{3} \text { hypergeom }\left(\left[1, \frac{-\frac{3 \mathrm{I}}{2}}{b d n}\right],\left[1-\frac{3 \mathrm{I}}{2 b d n}\right], \mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n}
$$

Result(type 8, 196 leaves):

$$
\begin{aligned}
& -\frac{x^{3}}{3}-\frac{2 \mathrm{I} x^{3}}{d b n\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}{2}\right)\right)\right)^{2}}-1\right)\right.}-\left(\frac{1}{2}\right) \\
& \left.\int \frac{-6 \mathrm{I} x^{2}}{d b n\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}{2}\right)\right)\right)^{2}}-1\right)\right.} \mathrm{d} x\right]
\end{aligned}
$$

Problem 57: Result more than twice size of optimal antiderivative.

$$
\int \frac{\cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{2}}{x} \mathrm{~d} x
$$

Optimal (type 3, 30 leaves, 3 steps):

$$
-\frac{\cot \left(a d+b d \ln \left(c x^{n}\right)\right)}{b d n}-\ln (x)
$$

Result(type 3, 62 leaves):

$$
-\frac{\cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)}{b d n}+\frac{\pi}{2 b d n}-\frac{\operatorname{arccot}\left(\cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)\right)}{b d n}
$$

Problem 58: Unable to integrate problem.

$$
\int \frac{\cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{2}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 140 leaves, 5 steps):

$$
\frac{1+\frac{\mathrm{I}}{b d n}}{x}+\frac{\mathrm{I}\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n x\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}-\frac{2 \operatorname{I} \text { hypergeom }\left(\left[1, \frac{\frac{\mathrm{I}}{2}}{b d n}\right],\left[1+\frac{\mathrm{I}}{2 b d n}\right], \mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b d n x}
$$

Result(type 8, 194 leaves):
$\frac{1}{x}-\frac{2 \mathrm{I}}{d b n x\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}{2}\right)\right)\right)^{2}}-1\right)\right.}-($

$$
\left.\left.\int \frac{2 \mathrm{I}}{\left.x^{2} b d n\left(\left(\mathrm{e}^{\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)\right.\right.}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right.}{2}\right)\right)\right)^{2}}-1\right) \mathrm{~d} x\right)
$$

Problem 60: Unable to integrate problem.

$$
\int(e x)^{m} \cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 5, 317 leaves, 6 steps):

$$
\begin{aligned}
& \frac{(\mathrm{I}(1+m)-b d n)(1+m+2 \mathrm{I} b d n)(e x)^{1+m}}{2 b^{2} d^{2} e(1+m) n^{2}}+\frac{(e x)^{1+m}\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)^{2}}{2 b d e n\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)^{2}} \\
& +\frac{\mathrm{I}(e x)^{1+m}\left(\frac{\mathrm{e}^{2 \mathrm{I} a d}(1+m-2 \mathrm{I} b d n)}{n}+\frac{\mathrm{e}^{4 \mathrm{I} a d}(1+m+2 \mathrm{I} b d n)\left(c x^{n}\right)^{2 \mathrm{I} b d}}{n}\right)}{2 b^{2} d^{2} e \mathrm{e}^{2 \mathrm{I} a d} n\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)} \\
& \quad-\frac{\mathrm{I}\left(-2 b^{2} d^{2} n^{2}+m^{2}+2 m+1\right)(e x)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{-\frac{\mathrm{I}}{2}(1+m)}{b d n}\right],\left[1-\frac{\mathrm{I}(1+m)}{2 b d n}\right], \mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{b^{2} d^{2} e(1+m) n^{2}}
\end{aligned}
$$

Result(type 8, 587 leaves):

$$
-\frac{\mathrm{I} x \mathrm{e}^{m\left(\ln (e)+\ln (x)-\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} e x)(-\operatorname{csgn}(\mathrm{I} e x)+\operatorname{csgn}(\mathrm{I} e))(-\operatorname{csgn}(\mathrm{I} e x)+\operatorname{csgn}(\mathrm{I} x))}{2}\right)}}{1+m}
$$

$$
-\left(\mathrm { I } x \mathrm { e } ^ { m ( \operatorname { l n } ( e ) + \operatorname { l n } ( x ) - \frac { \mathrm { I } \pi \operatorname { c s g n } ( \mathrm { I } e x ) ( - \operatorname { c s g n } ( \mathrm { I } e x ) + \operatorname { c s g n } ( \mathrm { I } e ) ) ( - \operatorname { c s g n } ( \mathrm { I } e x ) + \operatorname { c s g n } ( \mathrm { I } x ) ) } { 2 } ) } \left(2 \mathrm { I } \left(\mathrm{e}^{\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right.\right.}\right.}{2}\right)\right.}\right.\right.\right.
$$

$$
+m\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right.}{2}\right)\right)\right)^{2}, ~}\right.
$$

$$
\left.-\frac{\mathrm{e}^{m\left(\ln (e)+\ln (x)-\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} e x)(-\operatorname{csgn}(\mathrm{I} e x)+\operatorname{csgn}(\mathrm{I} e))(-\operatorname{csgn}(\mathrm{I} e x)+\operatorname{csgn}(\mathrm{I} x))}{2}\right)}\left(-2 b^{2} d^{2} n^{2}+m^{2}+2 m+1\right)}{d^{2} b^{2} n^{2}\left(\left(\mathrm{e}^{\left.\mathrm{I} d\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{\left.e \ln (x))+\operatorname{csgn}\left(\mathrm{I} e^{n \ln (x)}\right)\right)}\right.\right.}{2}\right)\right)\right)^{2}}-1\right)\right.} \mathrm{d} x\right)
$$

Problem 61: Unable to integrate problem.

$$
\int(e x)^{m} \cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{p} \mathrm{~d} x
$$

Optimal(type 6, 185 leaves, 5 steps):
$\frac{(e x)^{1+m}\left(1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)^{p}\left(\frac{-\mathrm{I}\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{1-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}}\right)^{p} A p p e l l F 1\left(\frac{-\frac{\mathrm{I}}{2}(1+m)}{b d n}, p,-p, 1-\frac{\mathrm{I}(1+m)}{2 b d n}, \mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d},-\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)}{e(1+m)\left(1+\mathrm{e}^{2 \mathrm{I} a d}\left(c x^{n}\right)^{2 \mathrm{I} b d}\right)^{p}}$
Result(type 8, 23 leaves):

$$
\int(e x)^{m} \cot \left(d\left(a+b \ln \left(c x^{n}\right)\right)\right)^{p} \mathrm{~d} x
$$

Problem 64: Unable to integrate problem.

$$
\int x \sec \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal(type 5, 72 leaves, 3 steps):

$$
\frac{2 \mathrm{e}^{\mathrm{I} a} x^{2}\left(c x^{n}\right)^{\mathrm{I} b} \text { hypergeom }\left(\left[1, \frac{1}{2}-\frac{\mathrm{I}}{b n}\right],\left[\frac{3}{2}-\frac{\mathrm{I}}{b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{2+\mathrm{I} b n}
$$

Result(type 8, 15 leaves):

$$
\int x \sec \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Problem 65: Unable to integrate problem.

$$
\int \sec \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal(type 5, 70 leaves, 3 steps):

$$
\frac{2 \mathrm{e}^{\mathrm{I} a} x\left(c x^{n}\right)^{\mathrm{I} b} \text { hypergeom }\left(\left[1, \frac{1}{2}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{3}{2}-\frac{\mathrm{I}}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{1+\mathrm{I} b n}
$$

Result(type 8, 13 leaves):

$$
\int \sec \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Problem 66: Unable to integrate problem.

$$
\int \frac{\sec \left(a+b \ln \left(c x^{n}\right)\right)}{x^{3}} \mathrm{~d} x
$$

Optimal(type 5, 72 leaves, 3 steps):

$$
-\frac{2 \mathrm{e}^{\mathrm{I} a}\left(c x^{n}\right)^{\mathrm{I} b} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{\mathrm{I}}{b n}\right],\left[\frac{3}{2}+\frac{\mathrm{I}}{b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2-\mathrm{I} b n) x^{2}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{\sec \left(a+b \ln \left(c x^{n}\right)\right)}{x^{3}} \mathrm{~d} x
$$

Problem 67: Unable to integrate problem.

$$
\int x^{2} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 72 leaves, 3 steps):

$$
\frac{4 \mathrm{e}^{2 \mathrm{I} a} x^{3}\left(c x^{n}\right)^{2 \mathrm{I} b} \text { hypergeom }\left(\left[2,1-\frac{3 \mathrm{I}}{2 b n}\right],\left[2-\frac{3 \mathrm{I}}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{3+2 \mathrm{I} b n}
$$

Result(type 8, 183 leaves):

$$
\begin{aligned}
& \left.\frac{22 x^{3}}{b n\left(\left(\mathrm{e}^{\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)}\right)^{2}+1\right)}+4\right)( \\
& \left.\int_{b n} \frac{-\frac{3 \mathrm{I}}{2} x^{2}}{\left(\left(\mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right.\right.\right.}\right)-\frac{\left.\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))+\operatorname{csgn}(\mathrm{I} c))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}\right)\right)\right)^{2}}{2}+1\right)} \mathrm{d} x\right)
\end{aligned}
$$

Problem 68: Unable to integrate problem.

$$
\int \frac{\sec \left(a+b \ln \left(c x^{n}\right)\right)^{2}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 72 leaves, 3 steps):

$$
-\frac{4 \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b} \text { hypergeom }\left(\left[2,1+\frac{\mathrm{I}}{2 b n}\right],\left[2+\frac{\mathrm{I}}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(1-2 \mathrm{I} b n) x}
$$

Result(type 8, 183 leaves):
$\frac{2 \mathrm{I}}{\left.\left.b n x\left(\left(\mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)\right.\right.}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2}+1\right)}+4($

$$
\left.\int \frac{\frac{\mathrm{I}}{2}}{\left.\left.x^{2} b n\left(\left(\mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{c}^{n} \ln (x)\right)(-\operatorname{csgn}(\mathrm{Ice}}{} \ln ^{\ln (x))+\operatorname{csgn}(\mathrm{Ic}))\left(-\operatorname{csgn}\left(\mathrm{Ic} \mathrm{c}^{n} \ln (x)\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}\right.\right.}\right)^{2}\right)\right)^{2}+1\right)} \mathrm{d} x\right)
$$

Problem 69: Unable to integrate problem.

$$
\int \sec \left(a+b \ln \left(c x^{n}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 5, 70 leaves, 3 steps):

$$
\frac{8 \mathrm{e}^{3 \mathrm{I} a} x\left(c x^{n}\right)^{3 \mathrm{I} b} \text { hypergeom }\left(\left[3, \frac{3}{2}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{5}{2}-\frac{\mathrm{I}}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{1+3 \mathrm{I} b n}
$$

Result(type 8, 487 leaves):
$-\left(\underset{\left.\left.\mathrm{I} x \mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right.}{2}\right)\right.}\right)\right)}{2}\right.$

$$
\left(\left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2} b n-b n n t h e r n}\right.\right.
$$

$$
-\mathrm{I}\left(\mathrm{e}^{\left.\left.\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2}-\mathrm{I}\right)\right) / . ~}\right.
$$

$$
\left(b^{2} n^{2}\left(\left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x))}\right)\right.}{2}\right)\right)\right)^{2}+1}\right)^{2}\right)+8\right)
$$

$$
\left.\int \frac{\mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)))}\right)}\right)\right.}{2}\left(b^{2} n^{2}+1\right)\right.\right.}}{8 b^{2} n^{2}\left(\left(\mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{\left.n \ln (x))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x))}\right)}\right)\right)\right)^{2}}\right.}{2}+1\right)\right.} \mathrm{d} x\right)\right.}\right\}
$$

Problem 70: Unable to integrate problem.

$$
\int \frac{\sec \left(a+b \ln \left(c x^{n}\right)\right)^{4}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 5, 72 leaves, 3 steps):

$$
-\frac{8 \mathrm{e}^{4 \mathrm{I} a}\left(c x^{n}\right)^{4 \mathrm{I} b} \text { hypergeom }\left(\left[4,2+\frac{\mathrm{I}}{b n}\right],\left[3+\frac{\mathrm{I}}{b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(1-2 \mathrm{I} b n) x^{2}}
$$

Result(type 8, 589 leaves):

$$
\begin{aligned}
& \left(4 \left(3 \mathrm { I } b ^ { 2 } n ^ { 2 } \left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2} .}\right.\right.\right. \\
& +b n\left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}{2}\right)\right)\right)^{4}+\mathrm{I} b^{2} n^{2}+{ }^{2} .}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left(3 b ^ { 3 } n ^ { 3 } x ^ { 2 } \left(\left(\mathrm{e}^{\left.\left.\left.\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}{2}\right)\right)\right)^{2}+1\right)^{3}\right)+16\right) .}\right.\right.\right. \\
& \left.\int \frac{\frac{\mathrm{I}}{6}\left(b^{2} n^{2}+1\right)}{b^{3} n^{3} x^{3}\left(\left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{\left.n \ln (x))+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}\right.\right.}{2}\right)\right)\right)^{2}}+1\right)\right.} \mathrm{dx}\right)
\end{aligned}
$$

Problem 71: Result more than twice size of optimal antiderivative.

$$
\int\left(-\left(b^{2} n^{2}+1\right) \sec \left(a+b \ln \left(c x^{n}\right)\right)+2 b^{2} n^{2} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{3}\right) \mathrm{d} x
$$

Optimal(type 3, 41 leaves, ? steps):

$$
-x \sec \left(a+b \ln \left(c x^{n}\right)\right)+b n x \sec \left(a+b \ln \left(c x^{n}\right)\right) \tan \left(a+b \ln \left(c x^{n}\right)\right)
$$

Result(type 3, 536 leaves):
$\left(-2 \mathrm{I} x\left(\left(\left(x^{n}\right)^{\mathrm{I} b}\right)^{3}\left(c^{\mathrm{I} b}\right)^{3} b n \mathrm{e}^{\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{3} \pi}{2}} \mathrm{e}^{-\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi}{2}} \mathrm{e}^{-\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e}^{\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right) \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e} \cdot \mathrm{I} a\right.\right.$ $-\left(x^{n}\right)^{\mathrm{I} b} c^{\mathrm{I} b} b n \mathrm{e}^{\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{3} \pi}{2}} \mathrm{e}^{-\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi}{2}} \mathrm{e}^{-\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e}^{\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right) \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e}^{\mathrm{I} a}$ $-\mathrm{I}\left(\left(x^{n}\right)^{\mathrm{I} b}\right)^{3}\left(c^{\mathrm{I} b}\right)^{3} \mathrm{e}^{\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{3} \pi}{2}} \mathrm{e}^{-\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi}{2}} \mathrm{e}^{-\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e}^{\frac{3 b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right) \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e}$ e $\mathrm{I} a$ $\left.\left.-\mathrm{I}\left(x^{n}\right)^{I b} c^{\mathrm{I} b} \mathrm{e}^{\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{3} \pi}{2}} \mathrm{e}^{-\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi}{2}} \mathrm{e}^{-\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{e}^{\frac{b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right) \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi \operatorname{csgn}(\mathrm{I} c)}{2}} \mathrm{I} a\right)\right) /$ $\left(\mathrm{e}^{-\mathrm{I}\left(\mathrm{I} b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{3} \pi-\mathrm{I} b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi-\mathrm{I} b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right)^{2} \pi \operatorname{csgn}(\mathrm{I} c)+\mathrm{I} b \operatorname{csgn}\left(\mathrm{I} c x^{n}\right) \operatorname{csgn}\left(\mathrm{I} x^{n}\right) \pi \operatorname{csgn}(\mathrm{I} c)-2 b \ln \left(x^{n}\right)-2 b \ln (c)-2 a\right)}+1\right)^{2}$

Problem 72: Unable to integrate problem.

$$
\int \sec \left(a+\frac{\mathrm{I} \ln \left(c x^{n}\right)}{n(-2+p)}\right)^{p} \mathrm{~d} x
$$

Optimal(type 3, 92 leaves, 3 steps):

$$
\frac{(2-p) x\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{\frac{2}{n(2-p)}}\right) \sec \left(a-\frac{\mathrm{I} \ln \left(c x^{n}\right)}{n(2-p)}\right)^{p}}{2 \mathrm{e}^{2 \mathrm{I} a}(1-p)\left(c x^{n}\right)^{\frac{2}{n(2-p)}}}
$$

Result(type 8, 24 leaves):

$$
\int \sec \left(a+\frac{\operatorname{Iln}\left(c x^{n}\right)}{n(-2+p)}\right)^{p} \mathrm{~d} x
$$

Problem 73: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{\sec \left(a+b \ln \left(c x^{n}\right)\right)}}{x} \mathrm{~d} x
$$

Optimal(type 4, 86 leaves, 3 steps):

$$
\frac{2 \sqrt{\cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right), \sqrt{2}\right) \sqrt{\cos \left(a+b \ln \left(c x^{n}\right)\right)} \sqrt{\sec \left(a+b \ln \left(c x^{n}\right)\right)}}{\cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right) b n}
$$

Result(type 4, 180 leaves):

$$
\begin{aligned}
& -\left(2 \sqrt { ( 2 \operatorname { c o s } ( \frac { a } { 2 } + \frac { b \operatorname { l n } ( c x ^ { n } ) } { 2 } ) ^ { 2 } - 1 ) \operatorname { s i n } ( \frac { a } { 2 } + \frac { b \operatorname { l n } ( c x ^ { n } ) } { 2 } ) ^ { 2 } } \sqrt { \operatorname { s i n } ( \frac { a } { 2 } + \frac { b \operatorname { l n } ( c x ^ { n } ) } { 2 } ) ^ { 2 } } \sqrt { - 2 \operatorname { c o s } ( \frac { a } { 2 } + \frac { b \operatorname { l n } ( c x ^ { n } ) } { 2 } ) ^ { 2 } + 1 } \operatorname { E l l i p t i c F } \left(\operatorname { c o s } \left(\frac{a}{2}\right.\right.\right. \\
& \left.\left.\left.\quad+\frac{b \ln \left(c x^{n}\right)}{2}\right), \sqrt{2}\right)\right) /\left(n \sqrt{-2 \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{4}+\sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}} \sin \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right) \sqrt{2 \cos \left(\frac{a}{2}+\frac{b \ln \left(c x^{n}\right)}{2}\right)^{2}-1 b}\right.
\end{aligned}
$$

Problem 74: Unable to integrate problem.

$$
\int x^{m} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 5, 91 leaves, 3 steps):

$$
\frac{8 \mathrm{e}^{3 \mathrm{I} a} x^{1+m}\left(c x^{n}\right)^{3 \mathrm{I} b} \text { hypergeom }\left(\left[3, \frac{-\mathrm{I}(1+m)+3 b n}{2 b n}\right],\left[\frac{-\mathrm{I}(1+m)+5 b n}{2 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{1+m+3 \mathrm{I} b n}
$$

Result(type 8, 577 leaves):

$$
\begin{aligned}
& -\left(x \mathrm{e}^{m \ln (x)} \mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))+\operatorname{csgn}(\mathrm{I} c))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))+\operatorname{csgn}(\mathrm{I}} \mathrm{e}^{n} \ln (x)\right)\right)}\right.\right.}{2}\right)\right)\left(I_{\mathrm{I}} .{ }^{2}\right)}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.+m\left(\mathrm{e}^{\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right.}{2}\right)\right.}\right)\right)^{2} \\
& +\left(\mathrm{e}^{\left.\left.\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}{2}\right)\right)\right)^{2}+m+1\right)\right) / . ~}\right.
\end{aligned}
$$

Problem 75: Unable to integrate problem.

$$
\int x^{m} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2} \mathrm{~d} x
$$

Optimal(type 5, 111 leaves, 3 steps):

$$
\frac{2 x^{1+m}\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{5 / 2} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{-2 \mathrm{I}-2 \mathrm{I} m+5 b n}{4 b n}\right],\left[\frac{-2 \mathrm{I}-2 \mathrm{I} m+9 b n}{4 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right) \sec \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2}}{2+2 m+5 \mathrm{I} b n}
$$

Result(type 8, 19 leaves):

$$
\int x^{m} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2} \mathrm{~d} x
$$

Problem 76: Unable to integrate problem.

$$
\int x^{m} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 5, 111 leaves, 3 steps):

$$
\frac{2 x^{1+m}\left(1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{3 / 2} \text { hypergeom }\left(\left[\frac{3}{2}, \frac{-2 \mathrm{I}-2 \mathrm{I} m+3 b n}{4 b n}\right],\left[\frac{-2 \mathrm{I}-2 \mathrm{I} m+7 b n}{4 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right) \sec \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}{2+2 m+3 \mathrm{I} b n}
$$

Result(type 8, 19 leaves):

$$
\int x^{m} \sec \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2} \mathrm{~d} x
$$

Problem 77: Unable to integrate problem.


Optimal(type 5, 111 leaves, 3 steps):

$$
\frac{2 x^{1+m} \text { hypergeom }\left(\left[-\frac{1}{2}, \frac{-2 \mathrm{I}-2 \mathrm{I} m-b n}{4 b n}\right],\left[\frac{-2 \mathrm{I}-2 \mathrm{I} m+3 b n}{4 b n}\right],-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2+2 m-\mathrm{I} b n) \sqrt{1+\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}} \sqrt{\sec \left(a+b \ln \left(c x^{n}\right)\right)}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{x^{m}}{\sqrt{\sec \left(a+b \ln \left(c x^{n}\right)\right)}} \mathrm{d} x
$$

Problem 78: Unable to integrate problem.

$$
\int x^{2} \csc \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal(type 5, 71 leaves, 3 steps):

$$
\frac{2 \mathrm{e}^{\mathrm{I} a} x^{3}\left(c x^{n}\right)^{\mathrm{I} b} \text { hypergeom }\left(\left[1, \frac{1}{2}-\frac{3 \mathrm{I}}{2 b n}\right],\left[\frac{3}{2}-\frac{3 \mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{3 \mathrm{I}-b n}
$$

Result(type 8, 17 leaves):

$$
\int x^{2} \csc \left(a+b \ln \left(c x^{n}\right)\right) \mathrm{d} x
$$

Problem 79: Unable to integrate problem.

$$
\int \csc \left(a+b \ln \left(c x^{n}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 69 leaves, 3 steps):

$$
-\frac{4 \mathrm{e}^{2 \mathrm{I} a} x\left(c x^{n}\right)^{2 \mathrm{I} b} \text { hypergeom }\left(\left[2,1-\frac{\mathrm{I}}{2 b n}\right],\left[2-\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{1+2 \mathrm{I} b n}
$$

Result(type 8, 178 leaves):
$-\frac{2 \mathrm{I} x}{\left.\left.b n\left(\left(\mathrm{e}^{\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)\right.\right.}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))+\operatorname{csgn}(\mathrm{I} c))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))}+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)\right.}\right.\right.}{2}\right)\right)\right)^{2}-1\right)}-4($

$$
\left.\int_{b n} \frac{-\frac{\mathrm{I}}{2}}{\left(\left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{\left.n \ln (x))+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}\right.\right.}{2}\right)\right)\right)^{2}}-1\right)\right.} \mathrm{d} x\right)
$$

Problem 80: Unable to integrate problem.

$$
\int \csc \left(a+b \ln \left(c x^{n}\right)\right)^{4} \mathrm{~d} x
$$

Optimal(type 5, 69 leaves, 3 steps):

$$
\frac{16 \mathrm{e}^{4 \mathrm{I} a} x\left(c x^{n}\right)^{4 \mathrm{I} b} \text { hypergeom }\left(\left[4,2-\frac{\mathrm{I}}{2 b n}\right],\left[3-\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{1+4 \mathrm{I} b n}
$$

Result(type 8, 587 leaves):

$$
\begin{aligned}
& \left(x \left(1 2 \mathrm { I } b ^ { 2 } n ^ { 2 } \left(\mathrm{e}^{\left.\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}\right.}{2}\right)\right)\right)^{2}\right)}\right.\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& -\mathrm{I}\left(\mathrm{e}^{\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right.}\right)}{2}\right)\right)\right)^{4}}\right.
\end{aligned}
$$

$$
\begin{aligned}
& +2 \mathrm{I}\left(\mathrm{e}^{\left.\left.\left.\mathrm{I}\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n \ln (x)}\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x)}\right)+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n \ln (x)}\right)\right)}{2}\right)\right)\right)^{2}-\mathrm{I}\right)\right) / . ~}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\int \frac{\frac{\mathrm{I}}{48}\left(4 b^{2} n^{2}+1\right)}{b^{3} n^{3}\left(\left(\mathrm{e}^{\left(a+b\left(\ln (c)+\ln \left(\mathrm{e}^{n} \ln (x)\right)-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n \ln (x))\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{n} \ln (x)\right)+\operatorname{csgn}(\mathrm{I} c)\right)\left(-\operatorname{csgn}\left(\mathrm{I} c \mathrm{e}^{\left.n \ln (x))+\operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{n} \ln (x)\right)\right)}\right.\right.}\right)}{2}\right)\right)^{2}}-1\right)\right.} \mathrm{d} x\right)
\end{aligned}
$$

Problem 81: Unable to integrate problem.

$$
\int x^{m} \csc \left(a+2 \ln \left(c x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 92 leaves, ? steps):

$$
\frac{x^{1+m} \csc \left(a+2 \ln \left(c x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)}{2(1+m)}-\frac{x^{1+m} \cot \left(a+2 \ln \left(c x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right) \csc \left(a+2 \ln \left(c x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)}{2 \sqrt{-(1+m)^{2}}}
$$

Result(type 8, 29 leaves):

$$
\int x^{m} \csc \left(a+2 \ln \left(c x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)^{3} \mathrm{~d} x
$$

Problem 83: Unable to integrate problem.

$$
\int \csc \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
\frac{2 x\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{5 / 2} \csc \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2} \operatorname{hypergeom}\left(\left[\frac{5}{2}, \frac{5}{4}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{9}{4}-\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{2+5 \mathrm{I} b n}
$$

Result(type 8, 15 leaves):

$$
\int \csc \left(a+b \ln \left(c x^{n}\right)\right)^{5 / 2} \mathrm{~d} x
$$

Problem 84: Unable to integrate problem.

$$
\int \frac{1}{\csc \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
\frac{2 x \text { hypergeom }\left(\left[-\frac{3}{2},-\frac{3}{4}-\frac{\mathrm{I}}{2 b n}\right],\left[\frac{1}{4}-\frac{\mathrm{I}}{2 b n}\right], \mathrm{e}^{\mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2-3 \mathrm{I} b n)\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{3 / 2} \csc \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{\csc \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 85: Unable to integrate problem.


Optimal(type 5, 110 leaves, 3 steps):

$$
\frac{2 x^{1+m} \text { hypergeom }\left(\left[-\frac{3}{2}, \frac{-2 \mathrm{I}-2 \mathrm{I} m-3 b n}{4 b n}\right],\left[\frac{-2 \mathrm{I}-2 \mathrm{I} m+b n}{4 b n}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)}{(2+2 m-3 \mathrm{I} b n)\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{3 / 2} \csc \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{x^{m}}{\csc \left(a+b \ln \left(c x^{n}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 86: Unable to integrate problem.

$$
\int \csc \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 95 leaves, 3 steps):

$$
x\left(1-\mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)^{p} \csc \left(a+b \ln \left(c x^{n}\right)\right)^{p} \operatorname{hypergeom}\left(\left[p, \frac{-\mathrm{I}+b n p}{2 b n}\right],\left[1-\frac{\mathrm{I}}{2 b n}+\frac{p}{2}\right], \mathrm{e}^{2 \mathrm{I} a}\left(c x^{n}\right)^{2 \mathrm{I} b}\right)
$$

$$
1+\mathrm{I} b n p
$$

Result(type 8, 15 leaves):

$$
\int \csc \left(a+b \ln \left(c x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Test results for the 41 problems in "4.7.6 $f^{\wedge}\left(a+b x+c x^{\wedge} 2\right)$ trig(d+e $\left.x+f x^{\wedge} 2\right)^{\wedge} n . t x t "$
Problem 1: Unable to integrate problem.

$$
\int F^{c(b x+a)} \sin (e x+d)^{n} \mathrm{~d} x
$$

Optimal(type 5, 97 leaves, 2 steps):

$$
-\frac{F^{c(b x+a)} \text { hypergeom }\left(\left[-n, \frac{-e n-\mathrm{I} b c \ln (F)}{2 e}\right],\left[1-\frac{n}{2}-\frac{\mathrm{I} b c \ln (F)}{2 e}\right], \mathrm{e}^{2 \mathrm{I}(e x+d)}\right) \sin (e x+d)^{n}}{\left(1-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)^{n}(\mathrm{I} e n-b c \ln (F))}
$$

Result(type 8, 20 leaves):

$$
\int F^{c(b x+a)} \sin (e x+d)^{n} \mathrm{~d} x
$$

Problem 2: Result more than twice size of optimal antiderivative.

$$
\int F^{c(b x+a)} \sin (e x+d)^{2} \mathrm{~d} x
$$

Optimal(type 3, 128 leaves, 2 steps):

$$
\frac{2 e^{2} F^{c(b x+a)}}{b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}-\frac{2 e F^{c(b x+a)} \cos (e x+d) \sin (e x+d)}{4 e^{2}+b^{2} c^{2} \ln (F)^{2}}+\frac{b c F^{c(b x+a)} \ln (F) \sin (e x+d)^{2}}{4 e^{2}+b^{2} c^{2} \ln (F)^{2}}
$$

Result(type 3, 267 leaves):

$$
\begin{aligned}
& \frac{1}{\left(1+\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}\right)^{2}}\left(-\frac{4 e \mathrm{e}^{c(b x+a) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{4 e^{2}+b^{2} c^{2} \ln (F)^{2}}+\frac{4 e \mathrm{e}^{c(b x+a) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{3}}{4 e^{2}+b^{2} c^{2} \ln (F)^{2}}+\frac{2 e^{2} \mathrm{e}^{c(b x+a) \ln (F)}}{b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}\right. \\
& \left.+\frac{2 e^{2} \mathrm{e}^{c(b x+a) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{4}}{b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}+\frac{4\left(e^{2}+b^{2} c^{2} \ln (F)^{2}\right) \mathrm{e}^{c(b x+a) \ln (F) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}}}{b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}\right)
\end{aligned}
$$

Problem 3: Unable to integrate problem.

$$
\int F^{c(b x+a)} \csc (e x+d)^{3} \mathrm{~d} x
$$

Optimal(type 5, 122 leaves, 2 steps):
$-\frac{F^{c(b x+a)} \cot (e x+d) \csc (e x+d)}{2 e}-\frac{b c F^{c(b x+a)} \csc (e x+d) \ln (F)}{2 e^{2}}$
$-\frac{\mathrm{e}^{\mathrm{I}(e x+d)} F^{c(b x+a)} \text { hypergeom }\left(\left[1, \frac{e-\mathrm{I} b c \ln (F)}{2 e}\right],\left[\frac{3}{2}-\frac{\mathrm{I} b c \ln (F)}{2 e}\right], \mathrm{e}^{2 \mathrm{I}(e x+d)}\right)(e+\mathrm{I} b c \ln (F))}{e^{2}}$
Result(type 8, 142 leaves):

$$
-\frac{\mathrm{I} \mathrm{e}^{c(b x+a) \ln (F)} \mathrm{e}^{\mathrm{I}(e x+d)}\left(\ln (F) b c\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+\mathrm{I}\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2} e-b c \ln (F)+\mathrm{I} e\right)}{e^{2}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}-1\right)^{2}}-8 \mathrm{I}\left(\int-\frac{\mathrm{e}^{c(b x+a) \ln (F)} \mathrm{e}^{\mathrm{I}(e x+d)}\left(e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}{8 e^{2}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}-1\right)} \mathrm{dx}\right)
$$

Problem 4: Unable to integrate problem.

$$
\int F^{c(b x+a)} \cos (e x+d)^{n} \mathrm{~d} x
$$

Optimal(type 5, 97 leaves, 2 steps):

$$
-\frac{F^{c(b x+a)} \cos (e x+d)^{n} \text { hypergeom }\left(\left[-n, \frac{-e n-\mathrm{I} b c \ln (F)}{2 e}\right],\left[1-\frac{n}{2}-\frac{\mathrm{I} b c \ln (F)}{2 e}\right],-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{\left(1+\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)^{n}(\mathrm{I} e n-b c \ln (F))}
$$

Result(type 8, 20 leaves):

$$
\int F^{c(b x+a)} \cos (e x+d)^{n} \mathrm{~d} x
$$

Problem 5: Unable to integrate problem.

$$
\int F^{c(b x+a)} \sec (e x+d)^{2} \mathrm{~d} x
$$

Optimal(type 5, 71 leaves, 1 step):

$$
\frac{4 \mathrm{e}^{2 \mathrm{I}(e x+d)} F^{c(b x+a)} \text { hypergeom }\left(\left[2,1-\frac{\mathrm{I} b c \ln (F)}{2 e}\right],\left[2-\frac{\mathrm{I} b c \ln (F)}{2 e}\right],-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{2 \mathrm{I} e+b c \ln (F)}
$$

Result(type 8, 71 leaves):

$$
\frac{2 \mathrm{I}^{c(b x+a) \ln (F)}}{e\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)}+4\left(\int \frac{-\frac{\mathrm{I}}{2} b c \ln (F) \mathrm{e}^{c(b x+a) \ln (F)}}{e\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)} \mathrm{d} x\right)
$$

Problem 6: Unable to integrate problem.

$$
\int F^{c(b x+a)} \sec (e x+d)^{3} \mathrm{~d} x
$$

Optimal(type 5, 126 leaves, 2 steps):


$$
+\frac{F^{c(b x+a)} \sec (e x+d) \tan (e x+d)}{2 e}
$$

Result(type 8, 139 leaves):

$$
-\frac{\mathrm{e}^{c(b x+a) \ln (F)} \mathrm{e}^{\mathrm{I}(e x+d)}\left(\ln (F) b c\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+\mathrm{I}\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2} e+b c \ln (F)-\mathrm{I} e\right)}{e^{2}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)^{2}}+8\left(\int \frac{\mathrm{e}^{c(b x+a) \ln (F)} \mathrm{e}^{\mathrm{I}(e x+d)}\left(e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}{8 e^{2}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)} \mathrm{dx}\right)
$$

Problem 7: Unable to integrate problem.

$$
\int F^{c(b x+a)} \sec (e x+d)^{4} \mathrm{~d} x
$$

Optimal(type 5, 128 leaves, 2 steps):
$-\frac{2 \mathrm{e}^{2 \mathrm{I}(e x+d)} F^{c(b x+a)} \operatorname{hypergeom}\left(\left[2,1-\frac{\mathrm{I} b c \ln (F)}{2 e}\right],\left[2-\frac{\mathrm{I} b c \ln (F)}{2 e}\right],-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)(2 \mathrm{I} e-b c \ln (F))}{3 e^{2}}-\frac{b c F^{c(b x+a)} \ln (F) \sec (e x+d)^{2}}{6 e^{2}}$

$$
+\frac{F^{c(b x+a)} \sec (e x+d)^{2} \tan (e x+d)}{3 e}
$$

Result(type 8, 204 leaves):
$\frac{1}{3 e^{3}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)^{3}}\left(\mathrm{Ie}^{c(b x+a) \ln (F)}\left(\ln (F)^{2} b^{2} c^{2}\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{4}+2 \ln (F)^{2} b^{2} c^{2}\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+2 \mathrm{I} \ln (F) b c e\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{4}+b^{2} c^{2} \ln (F)^{2}\right.\right.$
$\left.\left.+2 \operatorname{In}(F) b c e\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+12 e^{2}\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+4 e^{2}\right)\right)+16\left(\int \frac{-\frac{\mathrm{I}}{48} \mathrm{e}^{c(b x+a) \ln (F)} b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}{e^{3}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)} \mathrm{d} x\right)$

Problem 9: Unable to integrate problem.

$$
\int \mathrm{e}^{c(b x+a)} \tan (e x+d)^{3} \mathrm{~d} x
$$

Optimal(type 5, 168 leaves, 6 steps):
$\frac{\mathrm{Ie} e^{c(b x+a)}}{c b}-\frac{6 \mathrm{Ie}^{c(b x+a)} \text { hypergeom }\left(\left[1, \frac{-\frac{\mathrm{I}}{2} b c}{e}\right],\left[1-\frac{\mathrm{I} b c}{2 e}\right],-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{c b}+\frac{12 \mathrm{Ie} \mathrm{e}^{c(b x+a)} \text { hypergeom }\left(\left[2, \frac{-\frac{\mathrm{I}}{2} b c}{e}\right],\left[1-\frac{\mathrm{I} b c}{2 e}\right],-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{c b}$

$$
-\frac{8 \mathrm{Ie}^{c(b x+a)} \text { hypergeom }\left(\left[3, \frac{-\frac{\mathrm{I}}{2} b c}{e}\right],\left[1-\frac{\mathrm{I} b c}{2 e}\right],-\mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{c b}
$$

Result(type 8, 127 leaves):

$$
\frac{\mathrm{I} \mathrm{e}^{c(b x+a)}}{c b}-\frac{\mathrm{I} \mathrm{e}^{c(b x+a)}\left(2 \mathrm{I}\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2} e+b c\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+c b\right)}{e^{2}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)^{2}}+\mathrm{I}\left(\int-\frac{\mathrm{e}^{c(b x+a)}\left(-b^{2} c^{2}+2 e^{2}\right)}{e^{2}\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}+1\right)} \mathrm{d} x\right)
$$

Problem 10: Unable to integrate problem.

$$
\int \mathrm{e}^{c(b x+a)} \cot (e x+d)^{2} \mathrm{~d} x
$$

Optimal(type 5, 111 leaves, 5 steps):

$$
-\frac{\mathrm{e}^{c(b x+a)}}{c b}+\frac{4 \mathrm{e}^{c(b x+a)} \operatorname{hypergeom}\left(\left[1, \frac{-\frac{\mathrm{I}}{2} b c}{e}\right],\left[1-\frac{\mathrm{I} b c}{2 e}\right], \mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{c b}-\frac{4 \mathrm{e}^{c(b x+a)} \operatorname{hypergeom}\left(\left[2, \frac{-\frac{\mathrm{I}}{2} b c}{e}\right],\left[1-\frac{\mathrm{I} b c}{2 e}\right], \mathrm{e}^{2 \mathrm{I}(e x+d)}\right)}{c b}
$$

Result(type 8, 81 leaves):

$$
-\frac{\mathrm{e}^{c(b x+a)}}{c b}-\frac{2 \mathrm{I}^{c(b x+a)}}{e\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}-1\right)}-\left(\int \frac{-2 \mathrm{I} b c \mathrm{e}^{c(b x+a)}}{e\left(\left(\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2}-1\right)} \mathrm{d} x\right)
$$

Problem 13: Unable to integrate problem.

$$
\int F^{c(b x+a)}(f-f \cos (e x+d))^{n} \mathrm{~d} x
$$

Optimal(type 5, 101 leaves, 3 steps):

$$
-\frac{F^{c(b x+a)}(f-f \cos (e x+d))^{n} \text { hypergeom }\left(\left[-2 n,-n-\frac{\mathrm{I} b c \ln (F)}{e}\right],\left[1-n-\frac{\mathrm{I} b c \ln (F)}{e}\right], \mathrm{e}^{\mathrm{I}(e x+d)}\right)}{\left(1-\mathrm{e}^{\mathrm{I}(e x+d)}\right)^{2 n}(\mathrm{I} e n-b c \ln (F))}
$$

Result(type 8, 25 leaves):

$$
\int F^{c(b x+a)}(f-f \cos (e x+d))^{n} \mathrm{~d} x
$$

Problem 15: Result more than twice size of optimal antiderivative.

$$
\int \frac{F^{c(b x+a)}(f x)^{m}(e x \cos (e x+d)+(m+b c x \ln (F)) \sin (e x+d))}{x} \mathrm{~d} x
$$

Optimal(type 3, 22 leaves, 7 steps):

$$
F^{b c x+a c}(f x)^{m} \sin (e x+d)
$$

Result(type 3, 198 leaves):
$-\frac{\mathrm{I}}{2} F^{c(b x+a)}\left(f^{m} x^{m} \mathrm{e}^{\mathrm{I} e x} \mathrm{e}^{\mathrm{I} d} \mathrm{e}^{-\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x)^{3} m} \mathrm{e}^{\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x)^{2} \operatorname{csgn}(\mathrm{I} f) m} \mathrm{e}^{\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x)^{2} \operatorname{csgn}(\mathrm{I} x) m}-\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x) \operatorname{csgn}(\mathrm{I} f) \operatorname{csgn}(\mathrm{I} x) m\right.$

$$
\left.-f^{m} x^{m} \mathrm{e}^{-\mathrm{I} e x} \mathrm{e}^{-\mathrm{I} d} \mathrm{e}^{-\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x)^{3} m} \mathrm{e}^{\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x)^{2} \operatorname{csgn}(\mathrm{I} f) m} \mathrm{e}^{\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x)^{2} \operatorname{csgn}(\mathrm{I} x) m} \mathrm{e}^{-\frac{\mathrm{I}}{2} \pi \operatorname{csgn}(\mathrm{I} f x) \operatorname{csgn}(\mathrm{I} f) \operatorname{csgn}(\mathrm{I} x) m}\right)
$$

Problem 16: Result more than twice size of optimal antiderivative.

$$
\int F^{c(b x+a)}(e \cos (e x+d)+b c \ln (F) \sin (e x+d)) d x
$$

Optimal(type 3, 16 leaves, 1 step):

$$
F^{c(b x+a)} \sin (e x+d)
$$

Result(type 3, 292 leaves):
$\frac{e b c \ln (F) \mathrm{e}^{(b c x+a c) \ln (F)}}{e^{2}+b^{2} c^{2} \ln (F)^{2}}+\frac{2 e^{2} \mathrm{e}^{(b c x+a c) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e^{2}+b^{2} c^{2} \ln (F)^{2}}-\frac{e b c \ln (F) \mathrm{e}^{(b c x+a c) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}}{e^{2}+b^{2} c^{2} \ln (F)^{2}}$

$$
1+\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}
$$

$$
+\frac{\frac{e b c \ln (F) \mathrm{e}^{(b c x+a c) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}}{e^{2}+b^{2} c^{2} \ln (F)^{2}}+\frac{2 b^{2} c^{2} \ln (F)^{2} \mathrm{e}^{(b c x+a c) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e^{2}+b^{2} c^{2} \ln (F)^{2}}-\frac{e b c \ln (F) \mathrm{e}^{(b c x+a c) \ln (F)}}{e^{2}+b^{2} c^{2} \ln (F)^{2}}}{1+\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}}
$$

Problem 21: Unable to integrate problem.

$$
\int \frac{F^{b x+a} \cos (d x+c)}{e+e \sin (d x+c)} \mathrm{d} x
$$

Optimal(type 5, 77 leaves, 5 steps):

$$
\frac{\mathrm{I} F^{b x+a}}{b e \ln (F)}-\frac{2 \mathrm{I} F^{b x+a} \text { hypergeom }\left(\left[1, \frac{-\mathrm{I} b \ln (F)}{d}\right],\left[1-\frac{\mathrm{I} b \ln (F)}{d}\right], \mathrm{I} \mathrm{e}^{\mathrm{I}(d x+c)}\right)}{b e \ln (F)}
$$

Result (type 8, 53 leaves):

$$
\frac{\mathrm{I}^{(b x+a) \ln (F)}}{e b \ln (F)}+\int \frac{2 \mathrm{e}^{(b x+a) \ln (F)}}{e\left(\mathrm{e}^{\mathrm{I}(d x+c)}+\mathrm{I}\right)} \mathrm{d} x
$$

Problem 22: Unable to integrate problem.

$$
\int \frac{F^{b x+a} \cos (d x+c)}{e-e \sin (d x+c)} \mathrm{d} x
$$

Optimal(type 5, 77 leaves, 5 steps):

$$
-\frac{\mathrm{I} F^{b x+a}}{b e \ln (F)}+\frac{2 \mathrm{I} F^{b x+a} \text { hypergeom }\left(\left[1, \frac{-\mathrm{I} b \ln (F)}{d}\right],\left[1-\frac{\mathrm{I} b \ln (F)}{d}\right],-\mathrm{I} \mathrm{e}^{\mathrm{I}(d x+c)}\right)}{b e \ln (F)}
$$

Result(type 8, 53 leaves):

$$
-\frac{\mathrm{I} \mathrm{e}^{(b x+a) \ln (F)}}{e b \ln (F)}+\int \frac{2 \mathrm{e}^{(b x+a) \ln (F)}}{e\left(\mathrm{e}^{\mathrm{I}(d x+c)}-\mathrm{I}\right)} \mathrm{d} x
$$

Test results for the 247 problems in "4.7.7 Trig functions.txt"
Problem 13: Unable to integrate problem.

$$
\int \frac{\sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)^{3}}{-a^{2} x^{2}+1} \mathrm{~d} x
$$

Optimal(type 4, 46 leaves, 5 steps):

$$
-\frac{3 \mathrm{Si}\left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{4 a}+\frac{\operatorname{Si}\left(\frac{3 \sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{4 a}
$$

Result(type 8, 34 leaves):

$$
\int \frac{\sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)^{3}}{-a^{2} x^{2}+1} d x
$$

Problem 14: Unable to integrate problem.

$$
\int \frac{\sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{-a^{2} x^{2}+1} \mathrm{~d} x
$$

Optimal(type 4, 22 leaves, 2 steps):

$$
-\frac{\operatorname{Si}\left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{a}
$$

Result(type 8, 32 leaves):

$$
\int \frac{\sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{-a^{2} x^{2}+1} \mathrm{~d} x
$$

Problem 41: Result more than twice size of optimal antiderivative.

$$
\int-\cot (b x-c) \cot (b x+a) \mathrm{d} x
$$

Optimal(type 3, 37 leaves, 4 steps):

$$
x-\frac{\cot (a+c) \ln (-\sin (b x-c))}{b}+\frac{\cot (a+c) \ln (\sin (b x+a))}{b}
$$

Result(type 3, 148 leaves):

$$
x+\frac{\mathrm{I} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right) \mathrm{e}^{2 \mathrm{I}(a+c)}}{b\left(\mathrm{e}^{2 \mathrm{I}(a+c)}-1\right)}+\frac{\mathrm{I} \ln \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}-1\right)}{b\left(\mathrm{e}^{2 \mathrm{I}(a+c)}-1\right)}-\frac{\mathrm{I} \ln \left(-\mathrm{e}^{2 \mathrm{I}(a+c)}+\mathrm{e}^{2 \mathrm{I}(b x+a)}\right) \mathrm{e}^{2 \mathrm{I}(a+c)}}{b\left(\mathrm{e}^{2 \mathrm{I}(a+c)}-1\right)}-\frac{\mathrm{I} \ln \left(-\mathrm{e}^{2 \mathrm{I}(a+c)}+\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{b\left(\mathrm{e}^{2 \mathrm{I}(a+c)}-1\right)}
$$

Problem 42: Result more than twice size of optimal antiderivative.

$$
\int \sqrt{\sin (x) \tan (x)} \mathrm{d} x
$$

Optimal(type 3, 11 leaves, 2 steps):

$$
-2 \cot (x) \sqrt{\sin (x) \tan (x)}
$$

Result(type 3, 176 leaves):
$\frac{1}{4 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \sin (x)^{3}}\left(\sqrt{4}(\cos (x)-1)\left(4 \cos (x) \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+4 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+\ln (\right.\right.$

$$
\begin{aligned}
& \left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-\ln ( \\
& \left.\left.\left.\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)\right) \cos (x) \sqrt{-\frac{\cos (x)^{2}-1}{\cos (x)}}\right)
\end{aligned}
$$

Problem 43: Result more than twice size of optimal antiderivative.

$$
\int(\sin (x) \tan (x))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 3, 23 leaves, 3 steps):

$$
\frac{8 \csc (x) \sqrt{\sin (x) \tan (x)}}{3}-\frac{2 \sin (x) \sqrt{\sin (x) \tan (x)}}{3}
$$

Result(type 3, 586 leaves):

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+9 \cos (x)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-9 \cos (x)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right){ }^{3} / 2 \ln (
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+3 \ln (
$$

$$
\begin{aligned}
& \frac{1}{12 \sin (x)^{7}}\left(\sqrt { 4 } ( \operatorname { c o s } ( x ) - 1 ) ^ { 2 } \left(3 \cos (x)^{3}\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln ( \right.\right. \\
& \left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-3 \cos (x)^{3}\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln ( \\
& \left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+9\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{2} \ln ( \\
& \left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-9\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right) \cos ^{3}(x)^{2} \ln (2
\end{aligned}
$$

$\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2}-3 \ln ($
$\left.\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2}+4 \cos (x)^{3}+12 \cos (x)\right)(1$
$\left.+\cos (x))^{2}\left(-\frac{\cos (x)^{2}-1}{\cos (x)}\right)^{3 / 2}\right)$

Problem 45: Result more than twice size of optimal antiderivative.

$$
\int \frac{x \sin (x)}{(a+b \cos (x))^{2}} d x
$$

Optimal(type 3, 49 leaves, 3 steps):

$$
\frac{x}{b(a+b \cos (x))}-\frac{2 \arctan \left(\frac{\sqrt{a-b} \tan \left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b \sqrt{a-b} \sqrt{a+b}}
$$

Result(type 3, 153 leaves):

$$
\frac{2 x \mathrm{e}^{\mathrm{I} x}}{b\left(b \mathrm{e}^{2 \mathrm{I} x}+2 a \mathrm{e}^{\mathrm{I} x}+b\right)}-\frac{\mathrm{I} \ln \left(\mathrm{e}^{\mathrm{I} x}+\frac{a \sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{\sqrt{a^{2}-b^{2}} b}\right)}{\sqrt{a^{2}-b^{2}} b}+\frac{\mathrm{I} \ln \left(\mathrm{e}^{\mathrm{I} x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{\sqrt{a^{2}-b^{2}} b}\right)}{\sqrt{a^{2}-b^{2}} b}
$$

Problem 46: Result more than twice size of optimal antiderivative.

$$
\int \frac{x \sin (x)}{(a+b \cos (x))^{3}} \mathrm{~d} x
$$

Optimal(type 3, 74 leaves, 5 steps):

$$
-\frac{a \arctan \left(\frac{\sqrt{a-b} \tan \left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3 / 2} b(a+b)^{3 / 2}}+\frac{x}{2 b(a+b \cos (x))^{2}}+\frac{\sin (x)}{2\left(a^{2}-b^{2}\right)(a+b \cos (x))}
$$

Result(type 3, 249 leaves):
$\frac{\mathrm{I}\left(-2 \mathrm{I} a^{2} x \mathrm{e}^{2 \mathrm{I} x}+2 \mathrm{I} b^{2} x \mathrm{e}^{2 \mathrm{I} x}+a b \mathrm{e}^{3 \mathrm{I} x}+2 a^{2} \mathrm{e}^{2 \mathrm{I} x}+b^{2} \mathrm{e}^{2 \mathrm{I} x}+3 a b \mathrm{e}^{\mathrm{I} x}+b^{2}\right)}{b\left(b \mathrm{e}^{2 \mathrm{I} x}+2 a \mathrm{e}^{\mathrm{I} x}+b\right)^{2}\left(a^{2}-b^{2}\right)}-\frac{\mathrm{I} a \ln \left(\mathrm{e}^{\mathrm{I} x}+\frac{a \sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{\sqrt{a^{2}-b^{2}} b}\right)}{2 \sqrt{a^{2}-b^{2}}(a+b)(a-b) b}$

$$
+\frac{\mathrm{I} a \ln \left(\mathrm{e}^{\mathrm{I} x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{\sqrt{a^{2}-b^{2}} b}\right)}{2 \sqrt{a^{2}-b^{2}}(a+b)(a-b) b}
$$

Problem 47: Unable to integrate problem.

$$
\int x^{3} \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \mathrm{d} x
$$

Optimal(type 3, 139 leaves, 5 steps):
$-\frac{6 \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)}}{f^{4}}+\frac{3 x^{2} \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)}}{f^{2}}-\frac{6 x \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \tan (f x+e)}{f^{3}}$

$$
+\frac{x^{3} \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \tan (f x+e)}{f}
$$

Result(type 8, 31 leaves):

$$
\int x^{3} \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \mathrm{d} x
$$

Problem 48: Unable to integrate problem.

$$
\int x \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \mathrm{d} x
$$

Optimal (type 3, 66 leaves, 3 steps):

$$
\frac{\sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)}}{f^{2}}+\frac{x \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \tan (f x+e)}{f}
$$

Result (type 8, 29 leaves):

$$
\int x \sqrt{a-a \sin (f x+e)} \sqrt{c+c \sin (f x+e)} \mathrm{d} x
$$

Problem 51: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+B \sec (x)}{(a+a \cos (x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 71 leaves, 7 steps):

$$
\frac{2 B \operatorname{arctanh}\left(\frac{\sin (x) \sqrt{a}}{\sqrt{a+a \cos (x)}}\right)}{a^{3 / 2}}+\frac{(A-B) \sin (x)}{2(a+a \cos (x))^{3 / 2}}+\frac{(A-5 B) \operatorname{arctanh}\left(\frac{\sin (x) \sqrt{a} \sqrt{2}}{2 \sqrt{a+a \cos (x)}}\right) \sqrt{2}}{4 a^{3 / 2}}
$$

Result(type 3, 269 leaves):

$$
\left.\begin{array}{l}
-\frac{1}{4 a^{5} / 2 \cos \left(\frac{x}{2}\right) \sin \left(\frac{x}{2}\right) \sqrt{a \cos \left(\frac{x}{2}\right)^{2}}}\left(\sqrt{a \sin \left(\frac{x}{2}\right)^{2}}\right) A \sqrt{2} \ln \left(\frac{2\left(2 \sqrt{a} \sqrt{a \sin \left(\frac{x}{2}\right)^{2}}+2 a\right)}{\cos \left(\frac{x}{2}\right)}\right) \cos \left(\frac{x}{2}\right)^{2} a \\
-5 B \sqrt{2} \ln \left(\frac{2\left(2 \sqrt{a} \sqrt{a \sin \left(\frac{x}{2}\right)^{2}}+2 a\right)}{\cos \left(\frac{x}{2}\right)}\right) \cos \left(\frac{x}{2}\right)^{2} a+4 B \ln \left(-\frac{4\left(a \sqrt{2} \cos \left(\frac{x}{2}\right)-\sqrt{a} \sqrt{2} \sqrt{a \sin \left(\frac{x}{2}\right)^{2}}-2 a\right)}{2 \cos \left(\frac{x}{2}\right)-\sqrt{2}}\right) \cos \left(\frac{x}{2}\right)^{2} a \\
2 \cos \left(\frac{x}{2}\right)+\sqrt{2}
\end{array}\right) \cos \left(\frac{x}{2}\right)^{2} a+A \sqrt{2} \sqrt{\left.a \sin \left(\frac{x}{2}\right)^{2} \sqrt{a}-B \sqrt{2} \sqrt{a \sin \left(\frac{x}{2}\right)^{2}} \sqrt{a}\right)} .
$$

Problem 52: Result more than twice size of optimal antiderivative.

$$
\int \frac{x(b+a \cos (x))}{(a+b \cos (x))^{2}} d x
$$

Optimal(type 3, 24 leaves, 3 steps):

$$
\frac{\ln (a+b \cos (x))}{b}+\frac{x \sin (x)}{a+b \cos (x)}
$$

Result(type 3, 90 leaves):

$$
\frac{2 x \tan \left(\frac{x}{2}\right)+2 x \tan \left(\frac{x}{2}\right)^{3}}{\left(\tan \left(\frac{x}{2}\right)^{2}+1\right)\left(\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+a+b\right)}+\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+a+b\right)}{b}-\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2}+1\right)}{b}
$$

Problem 53: Result more than twice size of optimal antiderivative.

$$
\int \frac{-1+\frac{c^{2}}{d^{2}}+\sin (x)^{2}}{c+d \cos (x)} \mathrm{d} x
$$

Optimal(type 3, 14 leaves, 4 steps):

$$
\frac{c x}{d^{2}}-\frac{\sin (x)}{d}
$$

Result(type 3, 31 leaves):

$$
-\frac{2 \tan \left(\frac{x}{2}\right)}{d\left(\tan \left(\frac{x}{2}\right)^{2}+1\right)}+\frac{2 c \arctan \left(\tan \left(\frac{x}{2}\right)\right)}{d^{2}}
$$

Problem 56: Unable to integrate problem.

$$
\int(a \cos (d x+c)+b \sin (d x+c))^{n} \mathrm{~d} x
$$

Optimal(type 5, 128 leaves, 2 steps):
$-\frac{1}{d(n+1)\left(\frac{a \cos (d x+c)+b \sin (d x+c)}{\sqrt{a^{2}+b^{2}}}\right)^{n} \sqrt{\sin (c+d x-\arctan (a, b))^{2}}}\left(\cos (c+d x-\arctan (a, b))^{n+1}\right.$ hypergeom $\left(\left[\frac{1}{2}, \frac{n}{2}+\frac{1}{2}\right],\left[\frac{3}{2}+\frac{n}{2}\right]\right.$,

$$
\left.\left.\cos (c+d x-\arctan (a, b))^{2}\right)(a \cos (d x+c)+b \sin (d x+c))^{n} \sin (c+d x-\arctan (a, b))\right)
$$

Result(type 8, 21 leaves):

$$
\int(a \cos (d x+c)+b \sin (d x+c))^{n} \mathrm{~d} x
$$

Problem 61: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(a \cos (d x+c)+b \sin (d x+c))^{5}} \mathrm{~d} x
$$

Optimal(type 3, 146 leaves, 4 steps):

$$
-\frac{3 \operatorname{arctanh}\left(\frac{b \cos (d x+c)-a \sin (d x+c)}{\sqrt{a^{2}+b^{2}}}\right)}{8\left(a^{2}+b^{2}\right)^{5 / 2} d}+\frac{-b \cos (d x+c)+a \sin (d x+c)}{4\left(a^{2}+b^{2}\right) d(a \cos (d x+c)+b \sin (d x+c))^{4}}-\frac{3(b \cos (d x+c)-a \sin (d x+c))}{8\left(a^{2}+b^{2}\right)^{2} d(a \cos (d x+c)+b \sin (d x+c))^{2}}
$$

Result(type 3, 513 leaves):
$\frac{1}{d}\left(-\frac{1}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right) b-a\right)^{4}}\left(2\left(-\frac{\left(5 a^{4}+16 a^{2} b^{2}+8 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{8 a\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{3 b\left(a^{4}+16 a^{2} b^{2}+8 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{8 a^{2}\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}\right.\right.\right.$

$$
\begin{aligned}
& -\frac{\left(3 a^{6}-36 a^{4} b^{2}+56 a^{2} b^{4}+32 b^{6}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{8 a^{3}\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{b\left(15 a^{6}-114 a^{4} b^{2}-8 a^{2} b^{4}+16 b^{6}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{8 a^{4}\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)} \\
& -\frac{\left(3 a^{6}+84 a^{4} b^{2}-56 a^{2} b^{4}-32 b^{6}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{8 a^{3}\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{b\left(23 a^{4}-64 a^{2} b^{2}-24 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{8 a^{2}\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{\left(5 a^{4}-24 a^{2} b^{2}-8 b^{4}\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{8 a\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)} \\
& \left.\left.\left.+\frac{b\left(5 a^{2}+2 b^{2}\right)}{8\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}\right)\right)+\frac{3 \operatorname{arctanh}\left(\frac{2 a \tan \left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{4\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) \sqrt{a^{2}+b^{2}}}\right)}{2 \sqrt{a^{2}+b^{2}}}\right)
\end{aligned}
$$

Problem 69: Result more than twice size of optimal antiderivative.

$$
\int(a \cos (d x+c)+\mathrm{I} a \sin (d x+c))^{3} \mathrm{~d} x
$$

Optimal(type 3, 27 leaves, 1 step):

$$
\frac{-\frac{\mathrm{I}}{3}(a \cos (d x+c)+\mathrm{I} a \sin (d x+c))^{3}}{d}
$$

Result(type 3, 75 leaves):

$$
\frac{\frac{\mathrm{I} a^{3}\left(2+\sin (d x+c)^{2}\right) \cos (d x+c)}{3}-a^{3} \sin (d x+c)^{3}-\mathrm{I} a^{3} \cos (d x+c)^{3}+\frac{a^{3}\left(2+\cos (d x+c)^{2}\right) \sin (d x+c)}{3}}{d}
$$

Problem 79: Result more than twice size of optimal antiderivative.

$$
\int(\cot (x)+\csc (x))^{3} d x
$$

Optimal(type 3, 20 leaves, 4 steps):

$$
-\frac{2}{1-\cos (x)}-\ln (1-\cos (x))
$$

Result(type 3, 48 leaves):

$$
-\frac{\cot (x)^{2}}{2}-\ln (\sin (x))-\frac{3 \cos (x)^{3}}{2 \sin (x)^{2}}-\frac{3 \cos (x)}{2}-\ln (\csc (x)-\cot (x))-\frac{3}{2 \sin (x)^{2}}-\frac{\cot (x) \csc (x)}{2}
$$

Problem 88: Result more than twice size of optimal antiderivative.

$$
\int(-\cos (x)+\sec (x))^{7 / 2} \mathrm{~d} x
$$

Optimal(type 3, 57 leaves, 6 steps):
$-\frac{256 \csc (x) \sqrt{\sin (x) \tan (x)}}{35}+\frac{64 \sec (x) \sqrt{\sin (x) \tan (x)} \tan (x)}{35}-\frac{8 \sin (x) \sqrt{\sin (x) \tan (x)} \tan (x)^{2}}{7}-\frac{2 \sin (x)^{3} \sqrt{\sin (x) \tan (x)} \tan (x)^{2}}{7}$
Result(type 3, 602 leaves):

$$
-\frac{1}{70 \sin (x)^{11}}\left(( \operatorname { c o s } ( x ) - 1 ) ^ { 2 } \left(105\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{4} \ln (\right.\right.
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-105\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{4} \ln (
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+315 \cos (x)^{3}\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-315 \cos (x)^{3}\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+315\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{2} \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-315\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{2} \ln (
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)-20 \cos (x)^{6}+105 \cos (x)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-105 \cos (x)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+140 \cos (x)^{4}+420 \cos (x)^{2}-28\right) \cos (x)(1+\cos (x))^{2}(
$$

$$
\left.\left.-\frac{\cos (x)^{2}-1}{\cos (x)}\right)^{7 / 2}\right)
$$

Problem 89: Result more than twice size of optimal antiderivative.

$$
\int(-\cos (x)+\sec (x))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 3, 23 leaves, 4 steps):

$$
\frac{8 \csc (x) \sqrt{\sin (x) \tan (x)}}{3}-\frac{2 \sin (x) \sqrt{\sin (x) \tan (x)}}{3}
$$

Result(type 3, 583 leaves):

$$
\begin{aligned}
& \frac{1}{6 \sin (x)^{7}}\left(( \operatorname { c o s } ( x ) - 1 ) ^ { 2 } \left(3 \cos (x)^{3}\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln ( \right.\right. \\
& \left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-3 \cos (x)^{3}\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
\end{aligned}
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+9\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{2} \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-9\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \cos (x)^{2} \ln (
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+9 \cos (x)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)-9 \cos (x)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2} \ln (
$$

$$
\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)+3 \ln (
$$

$$
\left.-\frac{2\left(2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1\right)}{\sin (x)^{2}}\right)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2}-3 \ln (
$$

$$
\left.\left.-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)\left(-\frac{\cos (x)}{(1+\cos (x))^{2}}\right)^{3 / 2}+4 \cos (x)^{3}+12 \cos (x)\right)(1
$$

$\left.+\cos (x))^{2}\left(-\frac{\cos (x)^{2}-1}{\cos (x)}\right)^{3 / 2}\right)$

Problem 90: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{-\cos (x)+\sec (x)}} \mathrm{d} x
$$

Optimal(type 3, 40 leaves, 8 steps):

$$
\frac{\arctan (\sqrt{\cos (x)}) \sin (x)}{\sqrt{\cos (x)} \sqrt{\sin (x) \tan (x)}}-\frac{\operatorname{arctanh}(\sqrt{\cos (x)}) \sin (x)}{\sqrt{\cos (x)} \sqrt{\sin (x) \tan (x)}}
$$

Result(type 3, 104 leaves):
$-\frac{1}{2 \sin (x)}\left(\left(\arctan \left(\frac{1}{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}}\right)+\ln \left(-\frac{2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \cos (x)^{2}-\cos (x)^{2}-2 \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}}+2 \cos (x)-1}{\sin (x)^{2}}\right)\right)(1\right.$

$$
\left.+\cos (x)) \sqrt{-\frac{\cos (x)}{(1+\cos (x))^{2}}} \sqrt{\frac{1-\cos (x)^{2}}{\cos (x)}}\right)
$$

Problem 91: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(-\cos (x)+\sec (x))^{5 / 2}} d x
$$

Optimal(type 3, 67 leaves, 10 steps):

$$
\frac{3 \cot (x)}{16 \sqrt{\sin (x) \tan (x)}}-\frac{\cot (x) \csc (x)^{2}}{4 \sqrt{\sin (x) \tan (x)}}-\frac{3 \arctan (\sqrt{\cos (x)}) \sin (x)}{32 \sqrt{\cos (x)} \sqrt{\sin (x) \tan (x)}}+\frac{3 \operatorname{arctanh}(\sqrt{\cos (x)}) \sin (x)}{32 \sqrt{\cos (x)} \sqrt{\sin (x) \tan (x)}}
$$

Result(type 3, 453 leaves):


Problem 96: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+C \sin (x)}{b \cos (x)+c \sin (x)} \mathrm{d} x
$$

Optimal(type 3, 70 leaves, 3 steps):

$$
\frac{c C x}{b^{2}+c^{2}}-\frac{b C \ln (b \cos (x)+c \sin (x))}{b^{2}+c^{2}}-\frac{A \operatorname{arctanh}\left(\frac{c \cos (x)-b \sin (x)}{\sqrt{b^{2}+c^{2}}}\right)}{\sqrt{b^{2}+c^{2}}}
$$

Result(type 3, 149 leaves):

$$
\begin{aligned}
& \frac{C b \ln \left(\tan \left(\frac{x}{2}\right)^{2}+1\right)}{b^{2}+c^{2}}+\frac{2 C c \arctan \left(\tan \left(\frac{x}{2}\right)\right)}{b^{2}+c^{2}}-\frac{C b \ln \left(\tan \left(\frac{x}{2}\right)^{2} b-2 \tan \left(\frac{x}{2}\right) c-b\right)}{b^{2}+c^{2}}+\frac{2 \operatorname{arctanh}\left(\frac{2 \tan \left(\frac{x}{2}\right) b-2 c}{2 \sqrt{b^{2}+c^{2}}}\right) A b^{2}}{\left(b^{2}+c^{2}\right)^{3 / 2}} \\
& \quad+\frac{2 \operatorname{arctanh}\left(\frac{2 \tan \left(\frac{x}{2}\right) b-2 c}{2 \sqrt{b^{2}+c^{2}}}\right) A c^{2}}{\left(b^{2}+c^{2}\right)^{3 / 2}}
\end{aligned}
$$

Problem 102: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(2 a-2 a \cos (e x+d)+2 c \sin (e x+d))^{4}} \mathrm{~d} x
$$

Optimal(type 3, 202 leaves, 5 steps):

$$
\begin{aligned}
& \frac{a\left(5 a^{2}+3 c^{2}\right) \ln \left(a+c \cot \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{32 c^{7} e}+\frac{-c \cos (e x+d)-a \sin (e x+d)}{48 c^{2} e(a-a \cos (e x+d)+c \sin (e x+d))^{3}}+\frac{5\left(a c \cos (e x+d)+a^{2} \sin (e x+d)\right)}{96 c^{4} e(a-a \cos (e x+d)+c \sin (e x+d))^{2}} \\
& \quad+\frac{-c\left(15 a^{2}+4 c^{2}\right) \cos (e x+d)-a\left(15 a^{2}+4 c^{2}\right) \sin (e x+d)}{96 c^{6} e(a-a \cos (e x+d)+c \sin (e x+d))}
\end{aligned}
$$

Result(type 3, 415 leaves):

$$
\begin{array}{r}
-\frac{1}{384 e c^{4} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{3}}-\frac{5 a^{2}}{64 e c^{6} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}-\frac{3}{128 e c^{4} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}+\frac{a}{64 e c^{5} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}}-\frac{5 a^{3} \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{32 e c^{7}} \\
-\frac{3 a \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{32 e c^{5}}-\frac{a^{3}}{64 e c^{5}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{2}}-\frac{3 a}{128 e c^{3}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{2}}+\frac{128 e a^{3}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{2}}{128}-\frac{1}{128 e a^{3}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}-\frac{a^{3}}{384 e c^{4}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{3}}
\end{array}
$$

$$
\begin{aligned}
& -\frac{a}{128 e c^{2}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{3}}-\frac{1}{128 e a\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{3}}-\frac{c^{2}}{384 e a^{3}\left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)^{3}}+\frac{5 a^{3} \ln \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{32 e c^{7}} \\
& +\frac{3 a \ln \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{32 e c^{5}}
\end{aligned}
$$

Problem 107: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(2 a+2 b \cos (e x+d)-2 a \sin (e x+d))^{2}} \mathrm{~d} x
$$

Optimal(type 3, 73 leaves, 4 steps):

$$
-\frac{a \ln \left(a+b \tan \left(\frac{d}{2}+\frac{\pi}{4}+\frac{e x}{2}\right)\right)}{4 b^{3} e}+\frac{a \cos (e x+d)+b \sin (e x+d)}{4 b^{2} e(a+b \cos (e x+d)-a \sin (e x+d))}
$$

Result(type 3, 177 leaves):

$$
\begin{array}{r}
-\frac{a^{2}}{4 e b^{2}(a-b)\left(a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)-b \tan \left(\frac{e x}{2}+\frac{d}{2}\right)-a-b\right)}-\frac{1}{4 e(a-b)\left(a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)-b \tan \left(\frac{e x}{2}+\frac{d}{2}\right)-a-b\right)} \\
-\frac{a \ln \left(a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)-b \tan \left(\frac{e x}{2}+\frac{d}{2}\right)-a-b\right)}{4 e b^{3}}-\frac{a \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)-1\right)}{4 e b^{3}}
\end{array}
$$

Problem 108: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(a+b \cos (e x+d)+c \sin (e x+d))^{2}} \mathrm{~d} x
$$

Optimal(type 3, 116 leaves, 3 steps):

$$
\frac{2 a \arctan \left(\frac{c+(a-b) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{3 / 2} e}+\frac{c \cos (e x+d)-b \sin (e x+d)}{\left(a^{2}-b^{2}-c^{2}\right) e(a+b \cos (e x+d)+c \sin (e x+d))}
$$

Result(type 3, 423 leaves):

$$
-\frac{2 \tan \left(\frac{e x}{2}+\frac{d}{2}\right) a b}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+2 c \tan \left(\frac{e x}{2}+\frac{d}{2}\right)+a+b\right)\left(a^{3}-a^{2} b-a b^{2}-a c^{2}+b^{3}+c^{2} b\right)}
$$

$$
\begin{aligned}
& +\frac{2 \tan \left(\frac{e x}{2}+\frac{d}{2}\right) b^{2}}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+2 c \tan \left(\frac{e x}{2}+\frac{d}{2}\right)+a+b\right)\left(a^{3}-a^{2} b-a b^{2}-a c^{2}+b^{3}+c^{2} b\right)} \\
& +\frac{2 \tan \left(\frac{e x}{2}+\frac{d}{2}\right) c^{2}}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+2 c \tan \left(\frac{e x}{2}+\frac{d}{2}\right)+a+b\right)\left(a^{3}-a^{2} b-a b^{2}-a c^{2}+b^{3}+c^{2} b\right)} \\
& +\frac{2 a c}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+2 c \tan \left(\frac{e x}{2}+\frac{d}{2}\right)+a+b\right)\left(a^{3}-a^{2} b-a b^{2}-a c^{2}+b^{3}+c^{2} b\right)} \\
& +\frac{2 a \arctan \left(\frac{2(a-b) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right)}{e\left(a^{2}-b^{2}-c^{2}\right)^{3 / 2}}
\end{aligned}
$$

Problem 109: Result more than twice size of optimal antiderivative.

$$
\int \sqrt{2+3 \cos (e x+d)+5 \sin (e x+d)} \mathrm{d} x
$$

Optimal(type 4, 69 leaves, 2 steps):

$$
\frac{2 \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan \left(\frac{5}{3}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan \left(\frac{5}{3}\right)}{2}\right), \frac{\sqrt{510-30 \sqrt{34}}}{15}\right) \sqrt{2}+\sqrt{34}}{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan \left(\frac{5}{3}\right)}{2}\right) e}
$$

Result(type 4, 454 leaves):
$\frac{1}{17 \cos \left(e x+d+\arctan \left(\frac{3}{5}\right)\right) \sqrt{\sqrt{34} \sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)+2} e}\left(2 \sqrt{17} \sqrt{\frac{\sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)+1}{-\sqrt{34}+17}}\right.$

$$
\begin{aligned}
& \sqrt{-\frac{17\left(\sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\left(2 \sqrt{-\frac{17 \sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)+\sqrt{34}}{-\sqrt{34}+17}} \text { EllipticF } \sqrt{-\sqrt{34}+17},\right. \\
& \left.I \sqrt{\frac{-\sqrt{34}+17}{\sqrt{34}+17}}\right) \sqrt{34}+15 \sqrt{34} \sqrt{\frac{17 \sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}} \text { EllipticE } \sqrt{\frac{17 \sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right.}{\sqrt{34}+17}},
\end{aligned}
$$

$$
\begin{aligned}
& -34 \sqrt{\frac{17 \sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}} \text { EllipticF }\left(\sqrt{\left.\left.\frac{17 \sin \left(e x+d+\arctan \left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}, I \sqrt{\frac{\sqrt{34}+17}{-\sqrt{34}}+17}\right)\right)}\right)
\end{aligned}
$$

Problem 110: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{a+b \cos (e x+d)+c \sin (e x+d)}} \mathrm{d} x
$$

Optimal(type 4, 137 leaves, 2 steps):

$$
\frac{2 \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \sqrt{\frac{a+b \cos (e x+d)+c \sin (e x+d)}{a+\sqrt{b^{2}+c^{2}}}}}{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right) e \sqrt{a+b \cos (e x+d)+c \sin (e x+d)}}
$$

Result(type 4, 302 leaves):
$-(2(-a$

$$
\left.+\sqrt{b^{2}+c^{2}}\right)
$$



c) ) $\sqrt{\left.\frac{b^{2} \sin (e x+d-\arctan (-b, c))+c^{2} \sin (e x+d-\arctan (-b, c))+a \sqrt{b^{2}+c^{2}}}{\sqrt{b^{2}+c^{2}}} e\right)}$

Problem 111: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(a+b \cos (e x+d)+c \sin (e x+d))^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 428 leaves, 7 steps):

$$
\frac{2(c \cos (e x+d)-b \sin (e x+d))}{3\left(a^{2}-b^{2}-c^{2}\right) e(a+b \cos (e x+d)+c \sin (e x+d))^{3 / 2}}+\frac{8(a c \cos (e x+d)-a b \sin (e x+d))}{3\left(a^{2}-b^{2}-c^{2}\right)^{2} e \sqrt{a+b \cos (e x+d)+c \sin (e x+d)}}
$$

$$
+\frac{8 a \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\left.\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}\right) \sqrt{a+b \cos (e x+d)+c \sin (e x+d)}}\right.}{3 \cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right)\left(a^{2}-b^{2}-c^{2}\right)^{2} e \sqrt{\frac{a+b \cos (e x+d)+c \sin (e x+d)}{a+\sqrt{b^{2}+c^{2}}}}}
$$

$$
-\frac{2 \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\left.\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}\right) \sqrt{\frac{a+b \cos (e x+d)+c \sin (e x+d)}{a+\sqrt{b^{2}+c^{2}}}}} \underset{3 \cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (b, c)}{2}\right)\left(a^{2}-b^{2}-c^{2}\right) e \sqrt{a+b \cos (e x+d)+c \sin (e x+d)}}{ }\right.}{\frac{2}{2}}
$$

Result(type ?, 2966 leaves): Display of huge result suppressed!
Problem 117: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{b \cos (e x+d)+c \sin (e x+d)+\sqrt{b^{2}+c^{2}}}} d x
$$

Optimal(type 3, 75 leaves, 3 steps):

$$
\frac{\operatorname{arctanh}\left(\frac{\left(b^{2}+c^{2}\right)^{1 / 4} \sin (d+e x-\arctan (b, c)) \sqrt{2}}{2 \sqrt{\sqrt{b^{2}+c^{2}}+\cos (d+e x-\arctan (b, c)) \sqrt{b^{2}+c^{2}}}}\right) \sqrt{2}}{\left(b^{2}+c^{2}\right)^{1 / 4} e}
$$

Result(type 3, 171 leaves):


$$
\left(b^{2}+c^{2}\right)^{1 / 4} \cos (e x+d-\arctan (-b, c)) \sqrt{\frac{b^{2} \sin (e x+d-\arctan (-b, c))+c^{2} \sin (e x+d-\arctan (-b, c))+b^{2}+c^{2}}{\sqrt{b^{2}+c^{2}}}} e
$$

Problem 118: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(b \cos (e x+d)+c \sin (e x+d)+\sqrt{b^{2}+c^{2}}\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 137 leaves, 4 steps):

$$
\frac{\operatorname{arctanh}\left(\frac{\left(b^{2}+c^{2}\right)^{1 / 4} \sin (d+e x-\arctan (b, c)) \sqrt{2}}{2 \sqrt{\sqrt{b^{2}+c^{2}}+\cos (d+e x-\arctan (b, c)) \sqrt{b^{2}+c^{2}}}}\right) \sqrt{2}}{4\left(b^{2}+c^{2}\right)^{3 / 4} e}+\frac{-c \cos (e x+d)+b \sin (e x+d)}{2 e \sqrt{b^{2}+c^{2}}\left(b \cos (e x+d)+c \sin (e x+d)+\sqrt{b^{2}+c^{2}}\right)^{3 / 2}}
$$

Result(type 3, 349 leaves):
$-\left(\left(\sin (e x+d-\arctan (-b, c)) \operatorname{arctanh}\left(\frac{\sqrt{-\sin (e x+d-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+\sqrt{b^{2}+c^{2}}} \sqrt{2}}{2\left(b^{2}+c^{2}\right)^{1 / 4}}\right) \sqrt{2}\left(b^{2}+c^{2}\right)\right.\right.$

$$
+2 \sqrt{-\sin (e x+d-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+\sqrt{b^{2}+c^{2}}}\left(b^{2}+c^{2}\right)^{3 / 4}+\operatorname{arctanh}\left(\frac{\sqrt{-\sin (e x+d-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+\sqrt{b^{2}+c^{2}}} \sqrt{2\left(b^{2}+c^{2}\right)^{1 / 4}}}{2}\right) \sqrt{2} b^{2}
$$

$$
\left.\left.+\operatorname{arctanh}\left(\frac{\sqrt{-\sin (e x+d-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+\sqrt{b^{2}+c^{2}}} \sqrt{2}}{2\left(b^{2}+c^{2}\right)^{1 / 4}}\right) \sqrt{2} c^{2}\right) \sqrt{-\sqrt{b^{2}+c^{2}}(\sin (e x+d-\arctan (-b, c))-1)}\right) /\left(4 \left(b^{2}\right.\right.
$$

$$
\left.+c^{2}\right)^{7 / 4} \cos (e x+d-\arctan (-b, c)) \sqrt{\left.\frac{b^{2} \sin (e x+d-\arctan (-b, c))+c^{2} \sin (e x+d-\arctan (-b, c))+b^{2}+c^{2}}{\sqrt{b^{2}+c^{2}}} e\right)}
$$

Problem 121: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{a+c \sec (x)+\tan (x) b} \mathrm{~d} x
$$

Optimal(type 3, 91 leaves, 5 steps):

$$
\frac{a x}{a^{2}+b^{2}}+\frac{b \ln (c+a \cos (x)+b \sin (x))}{a^{2}+b^{2}}+\frac{2 a c \operatorname{arctanh}\left(\frac{b-(a-c) \tan \left(\frac{x}{2}\right)}{\sqrt{a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}+b^{2}\right) \sqrt{a^{2}+b^{2}-c^{2}}}
$$

Result(type 3, 413 leaves):

$$
-\frac{b \ln \left(\tan \left(\frac{x}{2}\right)^{2}+1\right)}{a^{2}+b^{2}}+\frac{2 a \arctan \left(\tan \left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}+\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-c \tan \left(\frac{x}{2}\right)^{2}-2 \tan \left(\frac{x}{2}\right) b-a-c\right) a b}{\left(a^{2}+b^{2}\right)(a-c)}
$$

Problem 122: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{\sec (e x+d)}}{\sqrt{a+b \sec (e x+d)+c \tan (e x+d)}} \mathrm{d} x
$$

Optimal(type 4, 145 leaves, 3 steps):

$$
\begin{aligned}
& \frac{1}{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (a, c)}{2}\right) e \sqrt{a+b \sec (e x+d)+c \tan (e x+d)}}\left(2 \sqrt { \operatorname { c o s } ( \frac { d } { 2 } + \frac { e x } { 2 } - \frac { \operatorname { a r c t a n } ( a , c ) } { 2 } ) ) ^ { 2 } } \operatorname { E l l i p t i c F } \left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (a, c)}{2}\right)\right.\right. \\
& \left.\sqrt{2+\sqrt{a^{2}+c^{2}}} \sqrt{\frac{\sec (e x+d)}{}} \sqrt{\frac{b+a \cos (e x+d)+c \sin (e x+d)}{b+\sqrt{a^{2}+c^{2}}}}\right)
\end{aligned}
$$

Result(type 4, 715 leaves):
-4 IEllipticF $\left(\sqrt{-\frac{\left(-\mathrm{I} a+\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c\right)(\mathrm{I} \sin (e x+d)+\cos (e x+d))}{\mathrm{I} a-\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c}}\right.$,

$$
\left.\sqrt{\frac{\left(\mathrm{I} a-\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c\right)\left(\mathrm{I} a-\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}+c\right)}{\left(-\mathrm{I} a+\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c\right)\left(-\mathrm{I} a+\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}+c\right)}}\right)
$$

$$
\sqrt{\frac{1}{\cos (e x+d)}} \sqrt{\frac{b+a \cos (e x+d)+c \sin (e x+d)}{\cos (e x+d)}}
$$

$$
\sqrt{\frac{\mathrm{I}\left(\sqrt{a^{2}-b^{2}+c^{2}} \cos (e x+d)+c \cos (e x+d)-a \sin (e x+d)+b \sin (e x+d)+\sqrt{a^{2}-b^{2}+c^{2}}+c\right)}{\left(-\mathrm{I} a+\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}+c\right)(\mathrm{I} \cos (e x+d)+\sin (e x+d)+\mathrm{I})}}
$$

$$
\sqrt{\frac{\mathrm{I}\left(a \sin (e x+d)-b \sin (e x+d)+\sqrt{a^{2}-b^{2}+c^{2}} \cos (e x+d)-c \cos (e x+d)+\sqrt{a^{2}-b^{2}+c^{2}}-c\right)}{\left(\mathrm{I} a-\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c\right)(\mathrm{I} \cos (e x+d)+\sin (e x+d)+\mathrm{I})}}
$$

$$
\begin{aligned}
& -\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-c \tan \left(\frac{x}{2}\right)^{2}-2 \tan \left(\frac{x}{2}\right) b-a-c\right) c b}{\left(a^{2}+b^{2}\right)(a-c)}+\frac{2 \arctan \left(\frac{2(a-c) \tan \left(\frac{x}{2}\right)-2 b}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) d a c}{\left(a^{2}+b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}}-\frac{2 \arctan \left(\frac{2(a-c) \tan \left(\frac{x}{2}\right)-2 b}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}-\left(a^{2}+b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}\right.}{(a)} \\
& +\frac{2 \arctan \left(\frac{2(a-c) \tan \left(\frac{x}{2}\right)-2 b}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) b^{2} a}{\left(a^{2}+b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}}-\frac{2 \arctan \left(\frac{2(a-c) \tan \left(\frac{x}{2}\right)-2 b}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) b^{2} c}{\left(a^{2}+b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}(a-c)}
\end{aligned}
$$

$$
\sqrt{-\frac{\left(-\mathrm{I} a+\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c\right)(\mathrm{I} \sin (e x+d)+\cos (e x+d))}{\mathrm{I} a-\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c}(\cos (e x+d)+1)^{2} \cos (e x+d)(\cos (e x+d)-1)^{2}\left(\mathrm{I} \sqrt{a^{2}-b^{2}+c^{2}} \sin (e x)\right.}
$$

$\left.\left.+d)-\mathrm{I} a \cos (e x+d)+\mathrm{I} b \cos (e x+d)-\mathrm{I} c \sin (e x+d)-\sqrt{a^{2}-b^{2}+c^{2}} \cos (e x+d)+c \cos (e x+d)-a \sin (e x+d)+b \sin (e x+d)\right)\right) /(e($

$$
\left.\left.-\mathrm{I} a+\mathrm{I} b+\sqrt{a^{2}-b^{2}+c^{2}}-c\right) \sin (e x+d)^{4}(b+a \cos (e x+d)+c \sin (e x+d))\right)
$$

Problem 123: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (e x+d)^{3 / 2}}{(a+b \sec (e x+d)+c \tan (e x+d))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 263 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{2 \sec (e x+d)^{3 / 2}(c \cos (e x+d)-a \sin (e x+d))(b+a \cos (e x+d)+c \sin (e x+d))}{\left(a^{2}-b^{2}+c^{2}\right) e(a+b \sec (e x+d)+c \tan (e x+d))^{3 / 2}}-\left(2 \sqrt { \operatorname { c o s } ( \frac { d } { 2 } + \frac { e x } { 2 } - \frac { \operatorname { a r c t a n } ( a , c ) } { 2 } ) ^ { 2 } } \operatorname { E l l i p t i c E } \left(\operatorname { s i n } \left(\frac{d}{2}\right.\right.\right. \\
& \left.\left.\left.+\frac{e x}{2}-\frac{\arctan (a, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^{2}+c^{2}}}{b+\sqrt{a^{2}+c^{2}}}}\right) \sec (e x+d)^{3 / 2}(b+a \cos (e x+d)+c \sin (e x+d))^{2}\right) /\left(\operatorname { c o s } ( \frac { d } { 2 } + \frac { e x } { 2 } - \frac { \operatorname { a r c t a n } ( a , c ) } { 2 } ) \left(a^{2}\right.\right. \\
& \left.\left.-b^{2}+c^{2}\right) e \sqrt{\frac{b+a \cos (e x+d)+c \sin (e x+d)}{b+\sqrt{a^{2}+c^{2}}}}(a+b \sec (e x+d)+c \tan (e x+d))^{3 / 2}\right)
\end{aligned}
$$

Result(type ?, 12426 leaves): Display of huge result suppressed!
Problem 124: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{a+b \cot (x)+c \csc (x)} \mathrm{d} x
$$

Optimal(type 3, 92 leaves, 5 steps):

$$
\frac{a x}{a^{2}+b^{2}}-\frac{b \ln (c+b \cos (x)+a \sin (x))}{a^{2}+b^{2}}+\frac{2 a c \operatorname{arctanh}\left(\frac{a-(b-c) \tan \left(\frac{x}{2}\right)}{\sqrt{a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}+b^{2}\right) \sqrt{a^{2}+b^{2}-c^{2}}}
$$

Result(type 3, 445 leaves):

$$
\frac{2 b \ln \left(\tan \left(\frac{x}{2}\right)^{2}+1\right)}{2 a^{2}+2 b^{2}}+\frac{4 a \arctan \left(\tan \left(\frac{x}{2}\right)\right)}{2 a^{2}+2 b^{2}}-\frac{2 \ln \left(\tan \left(\frac{x}{2}\right)^{2} b-c \tan \left(\frac{x}{2}\right)^{2}-2 a \tan \left(\frac{x}{2}\right)-b-c\right) b^{2}}{\left(2 a^{2}+2 b^{2}\right)(b-c)}
$$

$$
\begin{aligned}
& \left.+\frac{2 \ln \left(\tan \left(\frac{x}{2}\right)^{2} b-c \tan \left(\frac{x}{2}\right)^{2}-2 a \tan \left(\frac{x}{2}\right)-b-c\right) c b}{\left(2 a^{2}+2 b^{2}\right)(b-c)}+\frac{4 \arctan \left(\frac{2(b-c) \tan \left(\frac{x}{2}\right)-2 a}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) a b}{\left(2 a^{2}+2 b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}}+\frac{4 \arctan \left(\frac{2(b-c) \tan \left(\frac{x}{2}\right)-2 a}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) a\left(2 a^{2}+2 b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) a b^{2} \quad 4 \arctan \left(\frac{2(b-c) \tan \left(\frac{x}{2}\right)-2 a}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right) a c b \\
& -\frac{4 \arctan \left(\frac{2(b-c) \tan \left(\frac{x}{2}\right)-2 a}{\left(2 a^{2}+2 b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}}(b-c)\right.}{}+\frac{\left(2 a^{2}+2 b^{2}\right) \sqrt{-a^{2}-b^{2}+c^{2}}(b-c)}{(2)}
\end{aligned}
$$

Problem 127: Result more than twice size of optimal antiderivative.

$$
\int \frac{\csc (e x+d)^{3 / 2}}{(a+c \cot (e x+d)+b \csc (e x+d))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 263 leaves, 4 steps):
$-\frac{2 \csc (e x+d)^{3 / 2}(b+c \cos (e x+d)+a \sin (e x+d))(a \cos (e x+d)-c \sin (e x+d))}{\left(a^{2}-b^{2}+c^{2}\right) e(a+c \cot (e x+d)+b \csc (e x+d))^{3 / 2}}-\left(2 \csc (e x+d)^{3 / 2} \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right)^{2}} \operatorname{EllipticE}(\sin )\right.$ $\left.+d))^{2}\right) /\left(\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right)\left(a^{2}-b^{2}+c^{2}\right) e(a+c \cot (e x+d)+b \csc (e x+d))^{3 / 2} \sqrt{\frac{b+c \cos (e x+d)+a \sin (e x+d)}{b+\sqrt{a^{2}+c^{2}}}}\right)$
Result(type ?, 12233 leaves): Display of huge result suppressed!
Problem 128: Humongous result has more than 20000 leaves.


Optimal(type 4, 530 leaves, 8 steps):
$-\frac{2 \csc (e x+d)^{5 / 2}(b+c \cos (e x+d)+a \sin (e x+d))(a \cos (e x+d)-c \sin (e x+d))}{3\left(a^{2}-b^{2}+c^{2}\right) e(a+c \cot (e x+d)+b \csc (e x+d))^{5 / 2}}$

$$
\begin{aligned}
& +\frac{8 \csc (e x+d)^{5 / 2}(b+c \cos (e x+d)+a \sin (e x+d))^{2}(a b \cos (e x+d)-b c \sin (e x+d))}{3\left(a^{2}-b^{2}+c^{2}\right)^{2} e\left(a+c \cot (e x+d)+b \csc (e x+d)^{5 / 2}\right.}+\left(8 b \sqrt{\csc (e x+d)^{5 / 2}} \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right)^{2}} \mathrm{El}\right. \\
& \left.+c \cot (e x+d)+b \csc (e x+d))^{5 / 2} \sqrt{\frac{b+c \cos (e x+d)+a \sin (e x+d)}{b+\sqrt{a^{2}+c^{2}}}}\right)+\left(2 ^ { \operatorname { c s c } ( e x + d ) ^ { 5 / 2 } } \sqrt { \operatorname { c o s } ( \frac { d } { 2 } + \frac { e x } { 2 } - \frac { \operatorname { a r c t a n } ( c , a ) } { 2 } ) ^ { 2 } } \operatorname { E l l i p t i c F } \left(\operatorname { s i n } \left(\frac{d}{2}+\right.\right.\right.
\end{aligned}
$$

2) 

Result(type ?, 62958 leaves): Display of huge result suppressed!

Problem 129: Result more than twice size of optimal antiderivative.


Optimal(type 4, 145 leaves, 3 steps):

$$
\frac{2 \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right), \sqrt{2} \sqrt{\left.\frac{\sqrt{a^{2}+c^{2}}}{b+\sqrt{a^{2}+c^{2}}}\right) \sqrt{\frac{b+c \cos (e x+d)}{}+a \sin (e x+d)}}\right.}{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right) e \sqrt{a+c \cot (e x+d)+b \csc (e x+d)} \sqrt{\sin (e x+d)}}
$$

Result(type 4, 690 leaves):
1

$$
-4 \mathrm{I} \sqrt{\frac{b+c \cos (e x+d)+a \sin (e x+d)}{\sin (e x+d)}} \sqrt{\frac{\mathrm{I}\left(\sqrt{a^{2}-b^{2}+c^{2}} \cos (e x+d)-b \sin (e x+d)+c \sin (e x+d)-a \cos (e x+d)+\sqrt{a^{2}-b^{2}+c^{2}}-a\right)}{\left(-\mathrm{I} b+\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}-a\right)(\mathrm{I} \cos (e x+d)+\sin (e x+d)+\mathrm{I})}}
$$

$$
\sqrt{\frac{\mathrm{I}\left(b \sin (e x+d)-c \sin (e x+d)+\sqrt{a^{2}-b^{2}+c^{2}} \cos (e x+d)+a \cos (e x+d)+\sqrt{a^{2}-b^{2}+c^{2}}+a\right)}{\left(\mathrm{I} b-\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a\right)(\mathrm{I} \cos (e x+d)+\sin (e x+d)+\mathrm{I})}}
$$

$$
\sqrt{-\frac{\left(-\mathrm{I} b+\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a\right)(\mathrm{I} \sin (e x+d)+\cos (e x+d))}{\mathrm{I} b-\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a}}(\cos (e x+d)
$$

$$
+1)^{2} \text { EllipticF }\left(\sqrt{-\frac{\left(-\mathrm{I} b+\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a\right)(\mathrm{I} \sin (e x+d)+\cos (e x+d))}{\mathrm{I} b-\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a}},\right.
$$

$$
\left.\sqrt{\frac{\left(\mathrm{I} b-\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a\right)\left(\mathrm{I} b-\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}-a\right)}{\left(-\mathrm{I} b+\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}+a\right)\left(-\mathrm{I} b+\mathrm{I} c+\sqrt{a^{2}-b^{2}+c^{2}}-a\right)}}\right)(\cos (e x+d)-1)^{2}\left(\mathrm{I} \sqrt{a^{2}-b^{2}+c^{2}} \sin (e x+d)-\mathrm{I} b \cos (e x+d)\right.
$$

$$
\left.\left.+\mathrm{I} \cos (e x+d) c+\mathrm{I} \sin (e x+d) a-\sqrt{a^{2}-b^{2}+c^{2}} \cos (e x+d)-a \cos (e x+d)-b \sin (e x+d)+c \sin (e x+d)\right)\right) /(e(-\mathrm{I} b+\mathrm{I} c
$$

$$
\left.\left.+\sqrt{a^{2}-b^{2}+c^{2}}+a\right) \sin (e x+d)^{7 / 2}(b+c \cos (e x+d)+a \sin (e x+d))\right)
$$

Problem 130: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(a+c \cot (e x+d)+b \csc (e x+d))^{3 / 2} \sin (e x+d)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 263 leaves, 4 steps):
$-\frac{2(b+c \cos (e x+d)+a \sin (e x+d))(a \cos (e x+d)-c \sin (e x+d))}{\left(a^{2}-b^{2}+c^{2}\right) e(a+c \cot (e x+d)+b \csc (e x+d))^{3 / 2} \sin (e x+d)^{3 / 2}}$

$$
-\frac{2 \sqrt{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^{2}+c^{2}}}{b+\sqrt{a^{2}+c^{2}}}}\right)(b+c \cos (e x+d)+a \sin (e x+d))^{2}}{\cos \left(\frac{d}{2}+\frac{e x}{2}-\frac{\arctan (c, a)}{2}\right)\left(a^{2}-b^{2}+c^{2}\right) e(a+c \cot (e x+d)+b \csc (e x+d))^{3 / 2} \sin (e x+d)^{3 / 2} \sqrt{\frac{b+c \cos (e x+d)+a \sin (e x+d)}{b+\sqrt{a^{2}+c^{2}}}}}
$$

Result(type ?, 12223 leaves): Display of huge result suppressed!
Problem 134: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(\sec (x)^{2}-\tan (x)^{2}\right)^{3}} d x
$$

Optimal(type 1, 1 leaves, 2 steps):
Result(type 3, 3 leaves):
$x$

$$
\arctan (\tan (x))
$$

Problem 137: Result more than twice size of optimal antiderivative.

$$
\int \frac{d+e \sin (x)}{a+b \sin (x)+c \sin (x)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 206 leaves, 7 steps):
$\frac{\arctan \left(\frac{\left(2 c+\left(b-\sqrt{-4 a c+b^{2}}\right) \tan \left(\frac{x}{2}\right)\right) \sqrt{2}}{2 \sqrt{b^{2}-2 c(a+c)-b \sqrt{-4 a c+b^{2}}}}\right) \sqrt{2}\left(e+\frac{-b e+2 c d}{\sqrt{-4 a c+b^{2}}}\right)}{\sqrt{b^{2}-2 c(a+c)-b \sqrt{-4 a c+b^{2}}}}$
$+\frac{\arctan \left(\frac{\left(2 c+\left(b+\sqrt{-4 a c+b^{2}}\right) \tan \left(\frac{x}{2}\right)\right) \sqrt{2}}{2 \sqrt{b^{2}-2 c(a+c)+b \sqrt{-4 a c+b^{2}}}}\right) \sqrt{2}\left(e+\frac{b e-2 c d}{\sqrt{-4 a c+b^{2}}}\right)}{\sqrt{b^{2}-2 c(a+c)+b \sqrt{-4 a c+b^{2}}}}$
Result(type 3, 831 leaves):


Problem 138: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \sin (e x+d)}{\left(b^{2}+2 a b \sin (e x+d)+a^{2} \sin (e x+d)^{2}\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 224 leaves, 8 steps):

$$
\begin{aligned}
& \left.-\frac{\cos (e x+d)(b+a \sin (e x+d))}{2 e\left(b^{2}+2 a b \sin (e x+d)+a^{2} \sin (e x+d)^{2}\right)^{3 / 2}}-\frac{\operatorname{arctanh}\left(\frac{a+b \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{a^{2}\left(a^{2}-b^{2}\right)^{3 / 2} e\left(b^{2}+2 a b \sin (e x+d)+a^{2} \sin (e x+d)^{2}\right)^{3 / 2}}\right)\left(a b+a^{2} \sin (e x+d)\right)^{3}}{\sqrt{a^{2}-b^{2}}}\right) \\
& +\frac{b \cos (e x+d)\left(a b+a^{2} \sin (e x+d)\right)^{3}}{2\left(a^{2}-b^{2}\right) e\left(a^{3} b+a^{4} \sin (e x+d)\right)\left(b^{2}+2 a b \sin (e x+d)+a^{2} \sin (e x+d)^{2}\right)^{3 / 2}}
\end{aligned}
$$

Result (type 3, 737 leaves):
$-\left(-2 \arctan \left(\frac{b \cos (e x+d)-a \sin (e x+d)-b}{\sqrt{-a^{2}+b^{2}} \sin (e x+d)}\right) \cos (e x+d)^{2} \sin (e x+d) a^{4} b^{2}+\sqrt{-a^{2}+b^{2}} \cos (e x+d)^{3} a^{2} b^{3}-\sqrt{-a^{2}+b^{2}} \cos (e x+d)^{2} \sin (e x\right.$
$+d) a^{5}+2 \sqrt{-a^{2}+b^{2}} \cos (e x+d)^{2} \sin (e x+d) a^{3} b^{2}-6 \arctan \left(\frac{b \cos (e x+d)-a \sin (e x+d)-b}{\sqrt{-a^{2}+b^{2}} \sin (e x+d)}\right) \cos (e x+d)^{2} a^{3} b^{3}-3 \sqrt{-a^{2}+b^{2}} \cos (e x$
$+d)^{2} a^{4} b+6 \sqrt{-a^{2}+b^{2}} \cos (e x+d)^{2} a^{2} b^{3}+\sqrt{-a^{2}+b^{2}} \cos (e x+d) \sin (e x+d) a^{3} b^{2}-3 \sqrt{-a^{2}+b^{2}} \cos (e x+d) \sin (e x+d) a b^{4}$
$+2 \arctan \left(\frac{b \cos (e x+d)-a \sin (e x+d)-b}{\sqrt{-a^{2}+b^{2}} \sin (e x+d)}\right) \sin (e x+d) a^{4} b^{2}+6 \arctan \left(\frac{b \cos (e x+d)-a \sin (e x+d)-b}{\sqrt{-a^{2}+b^{2}} \sin (e x+d)}\right) \sin (e x+d) a^{2} b^{4}$
$-2 \sqrt{-a^{2}+b^{2}} \cos (e x+d) b^{5}+\sqrt{-a^{2}+b^{2}} \sin (e x+d) a^{5}+\sqrt{-a^{2}+b^{2}} \sin (e x+d) a^{3} b^{2}-6 \sqrt{-a^{2}+b^{2}} \sin (e x+d) a b^{4}$
$+6 \arctan \left(\frac{b \cos (e x+d)-a \sin (e x+d)-b}{\sqrt{-a^{2}+b^{2}} \sin (e x+d)}\right) a^{3} b^{3}+2 \arctan \left(\frac{b \cos (e x+d)-a \sin (e x+d)-b}{\sqrt{-a^{2}+b^{2}} \sin (e x+d)}\right) a b^{5}+3 \sqrt{-a^{2}+b^{2}} a^{4} b-5 \sqrt{-a^{2}+b^{2}} a^{2} b^{3}$
$\left.-2 \sqrt{-a^{2}+b^{2}} b^{5}\right) /\left(2 e \sqrt{-a^{2}+b^{2}}\left(a^{2}-b^{2}\right) b^{2}\left(-a^{2} \cos (e x+d)^{2}+2 a b \sin (e x+d)+a^{2}+b^{2}\right)^{3 / 2}\right)$

Problem 140: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \sec (e x+d)}{\left(b^{2}+2 a b \sec (e x+d)+a^{2} \sec (e x+d)^{2}\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 215 leaves, 8 steps):

$$
\begin{aligned}
\frac{a x}{b^{4}} & -\frac{\left(a^{2}-2 b^{2}\right)\left(2 a^{4}-a^{2} b^{2}+b^{4}\right) \arctan \left(\frac{\sqrt{a-b} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5 / 2} b^{4}(a+b)^{5 / 2} e}-\frac{a\left(3 a^{2}-5 b^{2}\right) \tan (e x+d)}{6 b^{2}\left(a^{2}-b^{2}\right) e(b+a \sec (e x+d))^{2}} \\
& -\frac{a\left(6 a^{4}-11 a^{2} b^{2}+11 b^{4}\right) \tan (e x+d)}{6 b^{3}\left(a^{2}-b^{2}\right)^{2} e(b+a \sec (e x+d))}-\frac{a^{4} \tan (e x+d)}{3 b e\left(a b+a^{2} \sec (e x+d)\right)^{3}}
\end{aligned}
$$

Result(type 3, 1117 leaves):

$$
\begin{aligned}
& \frac{2 a \arctan \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{e b^{4}}-\frac{2 a^{5} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{5}}{e b^{3}\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}+2 a b+b^{2}\right)} \\
& +\frac{a^{4} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{5}}{e b^{2}\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}+2 a b+b^{2}\right)} \\
& +\frac{4 a^{3} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{5}}{e b\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}+2 a b+b^{2}\right)}-\frac{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}+2 a b+b^{2}\right)}{3 a^{2} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{5}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{6 b a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{5}}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}+2 a b+b^{2}\right)}-\frac{4 a^{5} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{3}}{e b^{3}\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}(a-b)(a+b)} \\
& \left.+\frac{32 a^{3} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{3}}{3 e b\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}(a-b)(a+b)}-2 a^{5} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right] . \\
& a^{4} \tan \left(\frac{e x}{2}+\frac{d}{2}\right) \\
& e b^{2}\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}-2 a b+b^{2}\right) \\
& +\frac{4 a^{3} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e b\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}-2 a b+b^{2}\right)}+\frac{3 a^{2} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}-2 a b+b^{2}\right)} \\
& -\frac{6 b a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} a-\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2} b+a+b\right)^{3}\left(a^{2}-2 a b+b^{2}\right)}-\frac{2 \arctan \left(\frac{\tan \left(\frac{e x}{2}+\frac{d}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) a^{6}}{e b^{4}\left(a^{4}-2 a^{2} b^{2}+b^{4}\right) \sqrt{(a-b)(a+b)}} \\
& +\frac{5 \arctan \left(\frac{\tan \left(\frac{e x}{2}+\frac{d}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) a^{4}}{e b^{2}\left(a^{4}-2 a^{2} b^{2}+b^{4}\right) \sqrt{(a-b)(a+b)}}-\frac{3 \arctan \left(\frac{\tan \left(\frac{e x}{2}+\frac{d}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) a^{2}}{e\left(a^{4}-2 a^{2} b^{2}+b^{4}\right) \sqrt{(a-b)(a+b)}}+\frac{2 b^{2} \arctan \left(\frac{\tan \left(\frac{e x}{2}+\frac{d}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{e\left(a^{4}-2 a^{2} b^{2}+b^{4}\right) \sqrt{(a-b)(a+b)}}
\end{aligned}
$$

Problem 144: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+B \cos (x)}{a+b \cos (x)+\mathrm{I} b \sin (x)} \mathrm{d} x
$$

Optimal(type 3, 73 leaves, 1 step):

$$
\frac{(2 a A-b B) x}{2 a^{2}}+\frac{\mathrm{I} B \cos (x)}{2 a}+\frac{\mathrm{I}\left(2 a A b-B a^{2}-B b^{2}\right) \ln (a+b \cos (x)+\mathrm{I} b \sin (x))}{2 a^{2} b}+\frac{B \sin (x)}{2 a}
$$

Result(type 3, 152 leaves):

$$
\begin{gathered}
-\frac{\mathrm{I} \ln \left(\tan \left(\frac{x}{2}\right)-\mathrm{I}\right) A}{a}+\frac{\mathrm{I} \ln \left(\tan \left(\frac{x}{2}\right)-\mathrm{I}\right) b B}{2 a^{2}}+\frac{B}{a\left(\tan \left(\frac{x}{2}\right)-\mathrm{I}\right)}+\frac{\mathrm{I} B \ln \left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right)}{2 b}+\frac{\mathrm{I} \ln \left(\mathrm{I} a+\mathrm{I} b+a \tan \left(\frac{x}{2}\right)-\tan \left(\frac{x}{2}\right) b\right) A}{a} \\
-\frac{\mathrm{I} \ln \left(\mathrm{I} a+\mathrm{I} b+a \tan \left(\frac{x}{2}\right)-\tan \left(\frac{x}{2}\right) b\right) B}{2 b}-\frac{\mathrm{I} b \ln \left(\mathrm{I} a+\mathrm{I} b+a \tan \left(\frac{x}{2}\right)-\tan \left(\frac{x}{2}\right) b\right) B}{2 a^{2}}
\end{gathered}
$$

Problem 145: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+B \cos (x)}{a+b \cos (x)-\mathrm{I} b \sin (x)} \mathrm{d} x
$$

Optimal(type 3, 73 leaves, 1 step):

$$
\frac{(2 a A-b B) x}{2 a^{2}}-\frac{\mathrm{I} B \cos (x)}{2 a}-\frac{\mathrm{I}\left(2 a A b-B a^{2}-B b^{2}\right) \ln (a+b \cos (x)-\mathrm{I} b \sin (x))}{2 a^{2} b}+\frac{B \sin (x)}{2 a}
$$

Result(type 3, 283 leaves):

$$
\begin{aligned}
&-\frac{\mathrm{I} B \ln \left(\tan \left(\frac{x}{2}\right)-\mathrm{I}\right)}{2 b}+\frac{\mathrm{I} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) A}{-a+b}-\frac{\mathrm{I} b \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) A}{a(-a+b)} \\
&- \frac{\mathrm{I} a \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2 b(-a+b)}+\frac{\mathrm{I} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2(-a+b)}-\frac{\mathrm{I} b \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2 a(-a+b)} \\
& \quad+\frac{\mathrm{I} b^{2} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2 a^{2}(-a+b)}+\frac{\mathrm{I} \ln \left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right) A}{a}-\frac{\mathrm{I} \ln \left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right) b B}{2 a^{2}}+\frac{B}{a\left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right)}
\end{aligned}
$$

Problem 146: Result more than twice size of optimal antiderivative.

$$
\int \frac{B \cos (x)+C \sin (x)}{a+b \cos (x)+c \sin (x)} \mathrm{d} x
$$

Optimal(type 3, 113 leaves, 4 steps):

$$
\frac{(b B+C c) x}{b^{2}+c^{2}}+\frac{(B c-b C) \ln (a+b \cos (x)+c \sin (x))}{b^{2}+c^{2}}-\frac{2 a(b B+C c) \arctan \left(\frac{c+(a-b) \tan \left(\frac{x}{2}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}}
$$

Result(type 3, 823 leaves):
$\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+2 \tan \left(\frac{x}{2}\right) c+a+b\right) B a c}{\left(b^{2}+c^{2}\right)(a-b)}-\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+2 \tan \left(\frac{x}{2}\right) c+a+b\right) B b c}{\left(b^{2}+c^{2}\right)(a-b)}$

$$
\begin{aligned}
& -\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+2 \tan \left(\frac{x}{2}\right) c+a+b\right) C a b}{\left(b^{2}+c^{2}\right)(a-b)}+\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+2 \tan \left(\frac{x}{2}\right) c+a+b\right) C b^{2}}{\left(b^{2}+c^{2}\right)(a-b)} \\
& -\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) B a b}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}}+\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) B c^{2}}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}}-\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) C a c}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}} \\
& -\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) C b c}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}}-\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) c^{2} B a}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}(a-b)}+\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) c^{2} B b}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}(a-b)} \\
& +\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) c C a b}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}(a-b)}-\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) c C b^{2}}{\left(b^{2}+c^{2}\right) \sqrt{a^{2}-b^{2}-c^{2}}(a-b)}-\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2}+1\right) B c}{b^{2}+c^{2}}+\frac{C b \ln \left(\tan \left(\frac{x}{2}\right)^{2}+1\right)}{b^{2}+c^{2}} \\
& +\frac{2 B \arctan \left(\tan \left(\frac{x}{2}\right)\right) b}{b^{2}+c^{2}}+\frac{2 C c \arctan \left(\tan \left(\frac{x}{2}\right)\right)}{b^{2}+c^{2}}
\end{aligned}
$$

Problem 147: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+B \cos (x)+C \sin (x)}{(a+b \cos (x)+c \sin (x))^{2}} \mathrm{~d} x
$$

Optimal(type 3, 121 leaves, 4 steps):

$$
\frac{2(a A-b B-C c) \arctan \left(\frac{c+(a-b) \tan \left(\frac{x}{2}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{3 / 2}}+\frac{B c-b C+(A c-C a) \cos (x)-(A b-B a) \sin (x)}{\left(a^{2}-b^{2}-c^{2}\right)(a+b \cos (x)+c \sin (x))}
$$

Result(type 3, 328 leaves):

$$
\frac{2\left(-\frac{\left(a A b-A b^{2}-A c^{2}-B a^{2}+B a b+B c^{2}+C a c-C b c\right) \tan \left(\frac{x}{2}\right)}{a^{3}-a^{2} b-a b^{2}-a c^{2}+b^{3}+c^{2} b}+\frac{A a c-B b c-C a^{2}+C b^{2}}{a^{3}-a^{2} b-a b^{2}-a c^{2}+b^{3}+c^{2} b}\right)}{\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+2 \tan \left(\frac{x}{2}\right) c+a+b}
$$

$$
+\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) a A}{\left(a^{2}-b^{2}-c^{2}\right)^{3 / 2}}-\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) b B}{\left(a^{2}-b^{2}-c^{2}\right)^{3 / 2}}-\frac{2 \arctan \left(\frac{2(a-b) \tan \left(\frac{x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right) C c}{\left(a^{2}-b^{2}-c^{2}\right)^{3 / 2}}
$$

Problem 148: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+B \cos (x)+C \sin (x)}{a+b \cos (x)-\mathrm{I} b \sin (x)} \mathrm{d} x
$$

Optimal(type 3, 90 leaves, 1 step):

$$
\frac{(2 a A-b B+\mathrm{I} b C) x}{2 a^{2}}-\frac{\mathrm{I}\left(2 a A b-b^{2}(B-\mathrm{I} C)-a^{2}(B+\mathrm{I} C)\right) \ln (a+b \cos (x)-\mathrm{I} b \sin (x))}{2 a^{2} b}-\frac{(\mathrm{I} B+C)(\cos (x)+\mathrm{I} \sin (x))}{2 a}
$$

Result(type 3, 474 leaves):

$$
\begin{aligned}
& \frac{\ln \left(\tan \left(\frac{x}{2}\right)-\mathrm{I}\right) C}{2 b}-\frac{\mathrm{I} \ln \left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right) b B}{2 a^{2}}+\frac{a \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) C}{2 b(-a+b)}-\frac{\ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) C}{2(-a+b)} \\
& -\frac{b \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) C}{2 a(-a+b)}+\frac{b^{2} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) C}{2 a^{2}(-a+b)}-\frac{\mathrm{I} C}{a\left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right)} \\
& \quad+\frac{\mathrm{I} b^{2} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2 a^{2}(-a+b)}+\frac{\mathrm{I} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) A}{-a+b}-\frac{\mathrm{I} B \ln \left(\tan \left(\frac{x}{2}\right)-\mathrm{I}\right)}{2 b}+\frac{\mathrm{I} \ln \left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right) A}{a} \\
& \quad+\frac{\mathrm{I} \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2(-a+b)}-\frac{\mathrm{I} a \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2 b(-a+b)}+\frac{\mathrm{I}}{2\left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right)} \\
& \quad-\frac{\mathrm{I} b \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) A}{a(-a+b)}-\frac{\mathrm{I} b \ln \left(\mathrm{I} a+\mathrm{I} b-a \tan \left(\frac{x}{2}\right)+\tan \left(\frac{x}{2}\right) b\right) B}{2 a(-a+b)}-\frac{\ln \left(\tan \left(\frac{x}{2}\right)+\mathrm{I}\right) b C}{2 a}
\end{aligned}
$$

Problem 149: Result more than twice size of optimal antiderivative.

$$
\int \frac{b^{2}+c^{2}+a b \cos (x)+a c \sin (x)}{(a+b \cos (x)+c \sin (x))^{2}} \mathrm{~d} x
$$

Optimal(type 3, 23 leaves, 1 step):

$$
\frac{-c \cos (x)+b \sin (x)}{a+b \cos (x)+c \sin (x)}
$$

Result(type 3, 69 leaves):

$$
-\frac{2\left(-\frac{\left(a b-b^{2}-c^{2}\right) \tan \left(\frac{x}{2}\right)}{a-b}+\frac{a c}{a-b}\right)}{\tan \left(\frac{x}{2}\right)^{2} a-\tan \left(\frac{x}{2}\right)^{2} b+2 \tan \left(\frac{x}{2}\right) c+a+b}
$$

Problem 150: Result more than twice size of optimal antiderivative.

$$
\int(a+b \cos (x)+c \sin (x))^{5 / 2}(d+b e \cos (x)+c e \sin (x)) \mathrm{d} x
$$

Optimal(type 4, 414 leaves, 8 steps):
$-\frac{2(a+b \cos (x)+c \sin (x))^{5 / 2}(c e \cos (x)-b e \sin (x))}{7}-\frac{2(a+b \cos (x)+c \sin (x))^{3 / 2}(c(5 a e+7 d) \cos (x)-b(5 a e+7 d) \sin (x))}{35}$

$$
-\frac{2\left(c\left(56 a d+15 a^{2} e+25\left(b^{2}+c^{2}\right) e\right) \cos (x)-b\left(56 a d+15 a^{2} e+25\left(b^{2}+c^{2}\right) e\right) \sin (x)\right) \sqrt{a+b \cos (x)+c \sin (x)}}{105}
$$

$$
+\frac{1}{105 \cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right) \sqrt{\frac{a+b \cos (x)+c \sin (x)}{a+\sqrt{b^{2}+c^{2}}}}}\left(2 \left(161 a^{2} d+63\left(b^{2}+c^{2}\right) d+15 a^{3} e+145 a\left(b^{2}\right.\right.\right.
$$

$$
\left.\left.\left.+c^{2}\right) e\right) \sqrt{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \sqrt{a+b \cos (x)+c \sin (x)}\right)
$$

$$
-\frac{1}{105 \cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right) \sqrt{a+b \cos (x)+c \sin (x)}}\left(2 ( a ^ { 2 } - b ^ { 2 } - c ^ { 2 } ) \left(56 a d+15 a^{2} e+25\left(b^{2}\right.\right.\right.
$$

$$
\left.\left.\left.+c^{2}\right) e\right) \sqrt{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \sqrt{\frac{a+b \cos (x)+c \sin (x)}{a+\sqrt{b^{2}+c^{2}}}}\right)
$$

Result(type ?, 3501 leaves): Display of huge result suppressed!
Problem 151: Result more than twice size of optimal antiderivative.

$$
\int \sqrt{a+b \cos (x)+c \sin (x)}(d+b e \cos (x)+c e \sin (x)) \mathrm{d} x
$$

Optimal(type 4, 261 leaves, 6 steps):
$-\frac{2(c e \cos (x)-b e \sin (x)) \sqrt{a+b \cos (x)+c \sin (x)}}{3}$

$$
\begin{aligned}
& +\frac{2(a e+3 d) \sqrt{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \sqrt{a+b \cos (x)+c \sin (x)}}{3 \cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right) \sqrt{\frac{a+b \cos (x)+c \sin (x)}{a+\sqrt{b^{2}+c^{2}}}}} \\
& -\frac{2\left(a^{2}-b^{2}-c^{2}\right) e \sqrt{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\left.\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}\right) \sqrt{\frac{a+b \cos (x)+c \sin (x)}{a+\sqrt{b^{2}+c^{2}}}}}\right.}{3 \cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right) \sqrt{a+b \cos (x)+c \sin (x)}}
\end{aligned}
$$

Result(type 4, 1459 leaves):
$\left.\left(\sqrt{-\frac{\left(-b^{2} \sin (x-\arctan (-b, c))-c^{2} \sin (x-\arctan (-b, c))-a \sqrt{b^{2}+c^{2}}\right) \cos (x-\arctan (-b, c))^{2}}{\sqrt{b^{2}+c^{2}}}}\right)\left(\sqrt{b^{2}+c^{2}} b^{2} e+\sqrt{b^{2}+c^{2}} c^{2} e\right)\right)$

$$
\begin{aligned}
& -\frac{2 \sqrt{\cos (x-\arctan (-b, c))^{2}\left(\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+a\right)}}{3 \sqrt{b^{2}+c^{2}}} \\
& +\frac{1}{3 \sqrt{\cos (x-\arctan (-b, c))^{2}\left(\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+a\right)}} \\
& \left(2 \left(\frac{a}{\sqrt{b^{2}+c^{2}}}\right.\right.
\end{aligned}
$$

$$
-1)
$$

$$
\begin{aligned}
& \left.\left.\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}}, \sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)\right) \\
& -\frac{1}{3 \sqrt{b^{2}+c^{2}} \sqrt{\cos (x-\arctan (-b, c))^{2}\left(\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+a\right)}}\left(4 a \left(\frac{a}{\sqrt{b^{2}+c^{2}}}\right.\right. \\
& -1) \sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}} \sqrt{\frac{(-\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}} \sqrt{\frac{(\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{-a+\sqrt{b^{2}+c^{2}}}}(\sqrt{((\sqrt{2}})} \\
& \left.-\frac{a}{\sqrt{b^{2}+c^{2}}}-1\right) \text { EllipticE }\left(\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}}, \sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)+\text { EllipticF }\left(\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}},\right. \\
& \left.\left.\left.\sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)\right)\right)+\frac{1}{\sqrt{\cos (x-\arctan (-b, c))^{2}\left(\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+a\right)}}\left(2 ( a b ^ { 2 } e + a c ^ { 2 } e + b ^ { 2 } d + c ^ { 2 } d ) \left(\frac{a}{\sqrt{b^{2}+c^{2}}}\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& -1) \sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}} \sqrt{\frac{(-\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}} \sqrt{\frac{(\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{-a+\sqrt{b^{2}+c^{2}}}}(\sqrt{(\sqrt{2}})} \\
& \left.-\frac{a}{\sqrt{b^{2}+c^{2}}}-1\right) \text { EllipticE }\left(\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}}, \sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)+\text { EllipticF }\left(\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}},\right. \\
& \left.\left.\left.\sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)\right)\right)+\left(2 a d \sqrt { b ^ { 2 } + c ^ { 2 } } \left(\frac{a}{\sqrt{b^{2}+c^{2}}}\right.\right. \\
& -1) \\
& \sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}} \sqrt{\frac{(-\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}} \sqrt{\frac{(\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{-a+\sqrt{b^{2}+c^{2}}}} \text { EllipticF }( \\
& \left.\left.\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}}, \sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)\right) / \\
& \left.\sqrt{\left.-\frac{\left(-b^{2} \sin (x-\arctan (-b, c))-c^{2} \sin (x-\arctan (-b, c))-a \sqrt{b^{2}+c^{2}}\right) \cos (x-\arctan (-b, c))^{2}}{\sqrt{b^{2}+c^{2}}}\right)}\right) /\left(\sqrt{b^{2}+c^{2}} \cos (x-\arctan (-b,\right. \\
& \text { c) } \left.\sqrt{\frac{b^{2} \sin (x-\arctan (-b, c))+c^{2} \sin (x-\arctan (-b, c))+a \sqrt{b^{2}+c^{2}}}{\sqrt{b^{2}+c^{2}}}}\right)
\end{aligned}
$$

Problem 152: Result more than twice size of optimal antiderivative.

$$
\int \frac{d+b e \cos (x)+c e \sin (x)}{\sqrt{a+b \cos (x)+c \sin (x)}} \mathrm{d} x
$$

Optimal(type 4, 220 leaves, 5 steps):
$2 e \sqrt{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}}$ EllipticE $\left(\sin \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \sqrt{a+b \cos (x)+c \sin (x)}$

$$
\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right) \sqrt{\frac{a+b \cos (x)+c \sin (x)}{a+\sqrt{b^{2}+c^{2}}}}
$$

$$
+\frac{2(-a e+d) \sqrt{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right)^{2}} \text { EllipticF }\left(\sin \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \sqrt{\frac{a+b \cos (x)+c \sin (x)}{a+\sqrt{b^{2}+c^{2}}}}}{\cos \left(\frac{x}{2}-\frac{\arctan (b, c)}{2}\right) \sqrt{a+b \cos (x)+c \sin (x)}}
$$

Result(type 4, 776 leaves):
$\left(\sqrt{-\frac{\left(-b^{2} \sin (x-\arctan (-b, c))-c^{2} \sin (x-\arctan (-b, c))-a \sqrt{b^{2}+c^{2}}\right) \cos (x-\arctan (-b, c))^{2}}{\sqrt{b^{2}+c^{2}}}}\right)\left(2 d \sqrt{b^{2}+c^{2}}\left(\frac{a}{\sqrt{b^{2}+c^{2}}}\right.\right.$
$-1)$

$\left.\left.\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}}, \sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)\right) /$
$\sqrt{-\frac{\left(-b^{2} \sin (x-\arctan (-b, c))-c^{2} \sin (x-\arctan (-b, c))-a \sqrt{b^{2}+c^{2}}\right) \cos (x-\arctan (-b, c))^{2}}{\sqrt{b^{2}+c^{2}}}}$
$+\frac{1}{\sqrt{\cos (x-\arctan (-b, c))^{2}\left(\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}+a\right)}}\left(2\left(b^{2} e+c^{2} e\right)\left(\frac{a}{\sqrt{b^{2}+c^{2}}}\right.\right.$

$$
\begin{aligned}
& -1) \sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}} \sqrt{\frac{(-\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}} \sqrt{\frac{(\sin (x-\arctan (-b, c))+1) \sqrt{b^{2}+c^{2}}}{-a+\sqrt{b^{2}+c^{2}}}}((\sqrt{(\sqrt{2}})} \\
& \left.-\frac{a}{\sqrt{b^{2}+c^{2}}}-1\right) \text { EllipticE }\left(\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}}, \sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right)+\text { EllipticF }\left(\sqrt{\frac{-\sin (x-\arctan (-b, c)) \sqrt{b^{2}+c^{2}}-a}{-a+\sqrt{b^{2}+c^{2}}}},\right. \\
& \left.\sqrt{\frac{a-\sqrt{b^{2}+c^{2}}}{a+\sqrt{b^{2}+c^{2}}}}\right) \text { ) ) } /\left(\sqrt{b^{2}+c^{2}} \cos (x-\arctan (-b, c)) \sqrt{\frac{b^{2} \sin (x-\arctan (-b, c))+c^{2} \sin (x-\arctan (-b, c))+a \sqrt{b^{2}+c^{2}}}{\sqrt{b^{2}+c^{2}}}}\right)
\end{aligned}
$$

Problem 153: Result more than twice size of optimal antiderivative.

$$
\int \frac{A+B \cos (e x+d)+C \sin (e x+d)}{(a+c \sin (e x+d))^{4}} \mathrm{~d} x
$$

Optimal(type 3, 245 leaves, 10 steps):

$$
\begin{gathered}
\frac{\left(2 A a^{3}+3 A a c^{2}-4 C a^{2} c-C c^{3}\right) \arctan \left(\frac{c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{\sqrt{a^{2}-c^{2}}}\right)}{\left(a^{2}-c^{2}\right)^{7 / 2} e}-\frac{B}{3 c e(a+c \sin (e x+d))^{3}}+\frac{(A c-C a) \cos (e x+d)}{3\left(a^{2}-c^{2}\right) e(a+c \sin (e x+d))^{3}} \\
+\frac{\left(5 A a c-2 C a^{2}-3 C c^{2}\right) \cos (e x+d)}{6\left(a^{2}-c^{2}\right)^{2} e(a+c \sin (e x+d))^{2}}+\frac{\left(11 A a^{2} c+4 A c^{3}-2 C a^{3}-13 C a c^{2}\right) \cos (e x+d)}{6\left(a^{2}-c^{2}\right)^{3} e(a+c \sin (e x+d))}
\end{gathered}
$$

Result(type ?, 5050 leaves): Display of huge result suppressed!
Problem 155: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(a+\cos (d x+c) \sin (d x+c) b)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 161 leaves, 5 steps):

$$
\frac{2 b \cos (2 d x+2 c) \sqrt{2}}{\left(4 a^{2}-b^{2}\right) d \sqrt{2 a+b \sin (2 d x+2 c)}}-\frac{2 \sqrt{\sin \left(c+\frac{\pi}{4}+d x\right)^{2}} \operatorname{EllipticE}\left(\cos \left(c+\frac{\pi}{4}+d x\right), \sqrt{2} \sqrt{\frac{b}{2 a+b}}\right) \sqrt{2} \sqrt{2 a+b \sin (2 d x+2 c)}}{\sin \left(c+\frac{\pi}{4}+d x\right)\left(4 a^{2}-b^{2}\right) d \sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a+b}}}
$$

Result(type 4, 569 leaves):
$\frac{1}{b\left(4 a^{2}-b^{2}\right) \cos (2 d x+2 c) \sqrt{4 a+2 b \sin (2 d x+2 c)} d}\left(4\left(4 a^{2} \sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}} \sqrt{-\frac{(\sin (2 d x+2 c)-1) b}{2 a+b}} \sqrt{-\frac{(\sin (2 d x+2 c)+1) b}{2 a-b}}\right.\right.$

$$
\begin{aligned}
& \text { EllipticF }\left(\sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}}, \sqrt{\frac{2 a-b}{2 a+b}}\right) \\
& -\sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}} \sqrt{-\frac{(\sin (2 d x+2 c)-1) b}{2 a+b}} \sqrt{-\frac{(\sin (2 d x+2 c)+1) b}{2 a-b}} \operatorname{EllipticF}\left(\sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}}, \sqrt{\frac{2 a-b}{2 a+b}}\right) b^{2} \\
& -4 \sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}} \sqrt{-\frac{(\sin (2 d x+2 c)-1) b}{2 a+b}} \sqrt{-\frac{(\sin (2 d x+2 c)+1) b}{2 a-b}} \operatorname{EllipticE}\left(\sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}}, \sqrt{\frac{2 a-b}{2 a+b}}\right) a^{2} \\
& +\sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}} \sqrt{-\frac{(\sin (2 d x+2 c)-1) b}{2 a+b}} \sqrt{-\frac{(\sin (2 d x+2 c)+1) b}{2 a-b}} \operatorname{EllipticE}\left(\sqrt{\frac{2 a+b \sin (2 d x+2 c)}{2 a-b}}, \sqrt{\frac{2 a-b}{2 a+b}}\right) b^{2} \\
& \left.\left.-b^{2} \sin (2 d x+2 c)^{2}+b^{2}\right)\right)
\end{aligned}
$$

Problem 158: Attempted integration timed out after 120 seconds.

$$
\int \frac{\cos (a x)^{4}}{x^{2}(\cos (a x)+a x \sin (a x))^{2}} \mathrm{~d} x
$$

Optimal(type 4, 80 leaves, 6 steps):

$$
\frac{1}{x}+\frac{\cos (a x)^{2}}{a^{2} x^{3}}-\frac{2 \cos (a x)^{2}}{x}-2 a \operatorname{Si}(2 a x)-\frac{\cos (a x) \sin (a x)}{a x^{2}}-\frac{\cos (a x)^{3}}{a^{2} x^{3}(\cos (a x)+a x \sin (a x))}
$$

Result(type 1, 1 leaves):???
Problem 163: Result more than twice size of optimal antiderivative.

$$
\int \sqrt{c \tan (b x+a) \tan (2 b x+2 a)} \mathrm{d} x
$$

Optimal(type 3, 39 leaves, 3 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a)}{\sqrt{-c+c \sec (2 b x+2 a)}}\right) \sqrt{c}}{b}
$$

Result(type 3, 135 leaves):

$$
\sqrt{4} \sqrt{\frac{c\left(1-\cos (b x+a)^{2}\right)}{2 \cos (b x+a)^{2}-1}} \sin (b x+a) \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)
$$

$$
2 b(-1+\cos (b x+a))
$$

Problem 165: Result more than twice size of optimal antiderivative.

$$
\int(c \tan (b x+a) \tan (2 b x+2 a))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 3, 72 leaves, 5 steps):

$$
\frac{c^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a)}{\sqrt{-c+c \sec (2 b x+2 a)}}\right)}{b}+\frac{c^{2} \tan (2 b x+2 a)}{b \sqrt{-c+c \sec (2 b x+2 a)}}
$$

Result(type 3, 252 leaves):

$$
\begin{aligned}
& \frac{1}{b(2+\sqrt{2})(\sqrt{2}-2) \sin (b x+a)^{3}}\left(\sqrt { 2 } ( 2 \operatorname { c o s } ( b x + a ) ^ { 2 } - 1 ) \left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \sqrt{2} \cos (b x\right.\right. \\
& \quad+a) \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{\left.2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right)+\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{\left.2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right) \sqrt{2}}\right.} \begin{array}{l}
\quad-2 \cos (b x+a)\left(\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}\right)^{3 / 2}
\end{array}\right)
\end{aligned}
$$

Problem 166: Result more than twice size of optimal antiderivative.

$$
\int \cos (2 b x+2 a)(c \tan (b x+a) \tan (2 b x+2 a))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 3, 74 leaves, 6 steps):

$$
-\frac{3 c^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a)}{\sqrt{-c+c \sec (2 b x+2 a)}}\right)}{2 b}+\frac{c^{2} \sin (2 b x+2 a)}{2 b \sqrt{-c+c \sec (2 b x+2 a)}}
$$

Result(type 3, 517 leaves):

$$
\begin{aligned}
& -\frac{1}{b(2+\sqrt{2})(\sqrt{2}-2) \sin (b x+a)^{3}}\left(\sqrt { 2 } ( 2 \operatorname { c o s } ( b x + a ) ^ { 2 } - 1 ) \left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \sqrt{2} \cos (b x\right.\right. \\
& \quad+a) \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{\left.2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right)+\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{\left.2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right) \sqrt{2}}\right.} \begin{array}{l}
\quad-2 \cos (b x+a) \\
\left.\quad\left(\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}\right)^{3 / 2}\right)-\frac{1}{b(2+\sqrt{2})^{3}(\sqrt{2}-2)^{3} \sin (b x+a)^{3}}
\end{array}\right) .2 \sqrt{2}\left(2 \cos (b x+a)^{2}\right.
\end{aligned}
$$

$$
\left.\begin{array}{l}
-1)\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \sqrt{2} \cos (b x+a) \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)\right. \\
+\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{\left.\left.2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right) \sqrt{2}+4 \cos (b x+a)^{3}+2 \cos (b x+a)\right)}\right. \\
\left(\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}\right)^{3 / 2}
\end{array}\right)
$$

Problem 167: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (2 b x+2 a)^{4}}{\sqrt{c \tan (b x+a) \tan (2 b x+2 a)}} \mathrm{d} x
$$

Optimal(type 3, 154 leaves, 6 steps):
$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a) \sqrt{2}}{2 \sqrt{-c+c \sec (2 b x+2 a)}}\right) \sqrt{2}}{2 b \sqrt{c}}+\frac{14 \tan (2 b x+2 a)}{15 b \sqrt{-c+c \sec (2 b x+2 a)}}+\frac{\sec (2 b x+2 a)^{2} \tan (2 b x+2 a)}{5 b \sqrt{-c+c \sec (2 b x+2 a)}}$
$+\frac{\sqrt{-c+c \sec (2 b x+2 a)} \tan (2 b x+2 a)}{15 c b}$
Result(type 3, 979 leaves):
$\left(\sqrt{2} \sqrt{4}(-1+\cos (b x+a))\left(208 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{6}+120 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \cos (b x+a)^{6}\right.\right.$
$+120 \ln \left(-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\left.\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}+\cos (b x+a)+1\right)}\right.}{\sin (b x+a)^{2}}\right) \cos (b x+a)^{6}$

$$
+208 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{5}-200 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{4}
$$

$$
\begin{aligned}
& -180 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2}} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right) \cos (b x+a)^{4}-180 \ln ( \\
& \left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right) \cos (b x+a)^{4} \\
& -200 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{3}+60 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}
\end{aligned}
$$

$$
+90 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \cos (b x+a)^{2}+90 \ln (
$$

$$
\left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right) \cos (b x+a)^{2}+60 \cos (b x
$$

$$
+a) \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}-15 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)-15 \ln (
$$

$$
\left.\left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right)\right) /(120 b(-3
$$

$$
\left.+2 \sqrt{2})^{3}(3+2 \sqrt{2})^{3}\left(2 \cos (b x+a)^{2}-1\right)^{3} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \sqrt{\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}} \sin (b x+a)\right)
$$

[^5]$$
\int \frac{\sec (2 b x+2 a)^{3}}{\sqrt{c \tan (b x+a) \tan (2 b x+2 a)}} \mathrm{d} x
$$

Optimal(type 3, 112 leaves, 5 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a) \sqrt{2}}{2 \sqrt{-c+c \sec (2 b x+2 a)}}\right) \sqrt{2}}{2 b \sqrt{c}}+\frac{2 \tan (2 b x+2 a)}{3 b \sqrt{-c+c \sec (2 b x+2 a)}}+\frac{\sqrt{-c+c \sec (2 b x+2 a)} \tan (2 b x+2 a)}{3 c b}
$$

Result(type 3, 672 leaves):

$$
-\left(\sqrt { 2 } \sqrt { 4 } ( - 1 + \operatorname { c o s } ( b x + a ) ) \left(8 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{4}+12 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \cos (b x+a)^{4}\right.\right.
$$

$$
+12 \ln \left(-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right) \cos (b x+a)^{4}
$$

$$
+8 \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{3}-12 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \cos (b x+a)^{2}-12 \ln (
$$

$$
\left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right) \cos (b x+a)^{2}
$$

$$
+3 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)+3 \ln (
$$

$$
\left.\left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right)\right) /(24 b(-3
$$

$$
\left.+2 \sqrt{2})^{2}(3+2 \sqrt{2})^{2}\left(2 \cos (b x+a)^{2}-1\right)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \sqrt{\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}} \sin (b x+a)\right)
$$

Problem 169: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (2 b x+2 a)^{2}}{\sqrt{c \tan (b x+a) \tan (2 b x+2 a)}} \mathrm{d} x
$$

Optimal(type 3, 77 leaves, 4 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a) \sqrt{2}}{2 \sqrt{-c+c \sec (2 b x+2 a)}}\right) \sqrt{2}}{2 b \sqrt{c}}+\frac{\tan (2 b x+2 a)}{b \sqrt{-c+c \sec (2 b x+2 a)}}
$$

Result(type 3, 477 leaves):

$$
\begin{aligned}
& -\frac{1}{4 b \sqrt{\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}}\left(2 \cos (b x+a)^{2}-1\right)} \sqrt{2}\left(\sqrt { \frac { 2 \operatorname { c o s } ( b x + a ) ^ { 2 } - 1 } { ( \operatorname { c o s } ( b x + a ) + 1 ) ^ { 2 } } } \operatorname { a r c t a n h } \left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{\left.2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\right) \cos (b x+a)}\right.\right. \\
& \quad+\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \ln ( \\
& \left.\quad-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right) \cos (b x+a)
\end{aligned}
$$

$$
+\operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\ln (
$$

$$
\left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right)}{\sin (b x+a)^{2}}\right) \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}+4 \cos (b x}
$$

$$
+a)(\sin (b x+a))
$$

Problem 170: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sec (2 b x+2 a)}{\sqrt{c \tan (b x+a) \tan (2 b x+2 a)}} \mathrm{d} x
$$

Optimal(type 3, 46 leaves, 3 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a) \sqrt{2}}{2 \sqrt{-c+c \sec (2 b x+2 a)}}\right) \sqrt{2}}{2 b \sqrt{c}}
$$

Result(type 3, 235 leaves):
$\frac{1}{8 b \sin (b x+a) c}(\sqrt{2} \sqrt{4}(\cos (b x+a)$

$$
\begin{aligned}
& +1) \sqrt{\frac{c\left(1-\cos (b x+a)^{2}\right)}{2 \cos (b x+a)^{2}-1}} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}\left(\operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)+\ln ( \right. \\
& \left.-\frac{2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\left.\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}+\cos (b x+a)+1\right)}\right.}{\sin (b x+a)^{2}}\right)
\end{aligned}
$$

Problem 171: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(c \tan (b x+a) \tan (2 b x+2 a))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 117 leaves, 7 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a)}{\sqrt{-c+c \sec (2 b x+2 a)}}\right)}{b c^{3 / 2}}+\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{c} \tan (2 b x+2 a) \sqrt{2}}{2 \sqrt{-c+c \sec (2 b x+2 a)}}\right) \sqrt{2}}{8 b c^{3 / 2}}-\frac{\tan (2 b x+2 a)}{4 b(-c+c \sec (2 b x+2 a))^{3 / 2}}
$$

Result(type 3, 560 leaves):

$$
\begin{aligned}
& -\frac{1}{32 b\left(\frac{c \sin (b x+a)^{2}}{2 \cos (b x+a)^{2}-1}\right)^{3 / 2} \sin (b x+a)^{3}\left(\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}\right)^{3 / 2}}\left(\sqrt{2} \sqrt{4}(-1+\cos (b x+a))^{2}\right) 8 \sqrt{2} \cos (b x \\
& +a) \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)+2 \cos (b x+a) \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \\
& -5 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \cos (b x+a)-5 \ln ( \\
& 2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right) \\
& \sin (b x+a)^{2} \\
& -8 \operatorname{arctanh}\left(\frac{\sqrt{2} \cos (b x+a) \sqrt{4}(-1+\cos (b x+a))}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right) \sqrt{2}+5 \operatorname{arctanh}\left(\frac{\sqrt{4}\left(2 \cos (b x+a)^{2}-3 \cos (b x+a)+1\right)}{2 \sin (b x+a)^{2} \sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}}\right)+5 \ln ( \\
& 2\left(\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}} \cos (b x+a)^{2}-2 \cos (b x+a)^{2}-\sqrt{\frac{2 \cos (b x+a)^{2}-1}{(\cos (b x+a)+1)^{2}}}+\cos (b x+a)+1\right) \\
& \sin (b x+a)^{2}
\end{aligned}
$$

Problem 196: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{\sec (x)^{2}}{\sqrt{4-\sec (x)^{2}}} d x
$$

Optimal(type 3, 8 leaves, 2 steps):

$$
\arcsin \left(\frac{\tan (x) \sqrt{3}}{3}\right)
$$

Result(type 4, 102 leaves):
$-\frac{\sqrt{3} \sqrt{2} \sqrt{\frac{2 \cos (x)-1}{1+\cos (x)}} \sqrt{6} \sqrt{\frac{2 \cos (x)+1}{1+\cos (x)}}\left(\text { EllipticF }\left(\frac{\sqrt{3}(\cos (x)-1)}{\sin (x)}, \frac{1}{3}\right)-2 \text { EllipticPi }\left(\frac{\sqrt{3}(\cos (x)-1)}{\sin (x)}, \frac{1}{3}, \frac{1}{3}\right)\right) \sin (x)^{2}}{}$

$$
9 \sqrt{\frac{4 \cos (x)^{2}-1}{\cos (x)^{2}}} \cos (x)(\cos (x)-1)
$$

Problem 199: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \cot (x))^{3} \csc (x)^{2}}{c+d \cot (x)} \mathrm{d} x
$$

Optimal(type 3, 74 leaves, 3 steps):

$$
-\frac{b(-a d+c b)^{2} \cot (x)}{d^{3}}+\frac{(-a d+c b)(a+b \cot (x))^{2}}{2 d^{2}}-\frac{(a+b \cot (x))^{3}}{3 d}+\frac{(-a d+c b)^{3} \ln (c+d \cot (x))}{d^{4}}
$$

Result(type 3, 201 leaves):

$$
\begin{aligned}
& -\frac{\ln (\tan (x) c+d) a^{3}}{d}+\frac{3 \ln (\tan (x) c+d) a^{2} b c}{d^{2}}-\frac{3 \ln (\tan (x) c+d) a b^{2} c^{2}}{d^{3}}+\frac{\ln (\tan (x) c+d) b^{3} c^{3}}{d^{4}}-\frac{b^{3}}{3 d \tan (x)^{3}}+\frac{\ln (\tan (x)) a^{3}}{d} \\
& \quad-\frac{3 \ln (\tan (x)) a^{2} b c}{d^{2}}+\frac{3 \ln (\tan (x)) a b^{2} c^{2}}{d^{3}}-\frac{\ln (\tan (x)) b^{3} c^{3}}{d^{4}}-\frac{3 b a^{2}}{d \tan (x)}+\frac{3 b^{2} a c}{d^{2} \tan (x)}-\frac{b^{3} c^{2}}{d^{3} \tan (x)}-\frac{3 b^{2} a}{2 d \tan (x)^{2}}+\frac{b^{3} c}{2 d^{2} \tan (x)^{2}}
\end{aligned}
$$

Problem 205: Result more than twice size of optimal antiderivative.

$$
\int \mathrm{e}^{n \sin (b x+a)} \sin (2 b x+2 a) \mathrm{d} x
$$

Optimal(type 3, 41 leaves, 4 steps):

$$
-\frac{2 \mathrm{e}^{n \sin (b x+a)}}{b n^{2}}+\frac{2 \mathrm{e}^{n \sin (b x+a)} \sin (b x+a)}{b n}
$$

Result(type 3, 103 leaves):

$$
-\frac{\mathrm{I}^{n \sin (b x) \cos (a)+n \cos (b x) \sin (a)} \mathrm{e}^{\mathrm{I} b x} \mathrm{e}^{\mathrm{I} a}}{b n}+\frac{\mathrm{I} \mathrm{e}^{n \sin (b x) \cos (a)+n \cos (b x) \sin (a)} \mathrm{e}^{-\mathrm{I} b x} \mathrm{e}^{-\mathrm{I} a}}{b n}-\frac{2 \mathrm{e}^{n(\sin (b x) \cos (a)+\cos (b x) \sin (a))}}{b n^{2}}
$$

Problem 231: Result more than twice size of optimal antiderivative.

$$
\int \cot (x)^{3} \csc (x) \sqrt{1+\csc (x)} \mathrm{d} x
$$

Optimal(type 3, 17 leaves, 6 steps):

$$
\frac{4(1+\csc (x))^{5 / 2}}{5}-\frac{2(1+\csc (x))^{7 / 2}}{7}
$$

Result(type 3, 37 leaves):
$-\frac{2 \sqrt{\frac{1+\sin (x)}{\sin (x)}}\left(9 \cos (x)^{2} \sin (x)+13 \cos (x)^{2}-8 \sin (x)-8\right)}{35 \sin (x)^{3}}$

Problem 232: Unable to integrate problem.

$$
\int \sqrt{\csc (x)}(\cos (x) x-4 \sec (x) \tan (x)) \mathrm{d} x
$$

Optimal(type 3, 16 leaves, 8 steps):

$$
-\frac{4 \sec (x)}{\csc (x)^{3 / 2}}+\frac{2 x}{\sqrt{\csc (x)}}
$$

Result(type 8, 18 leaves):

$$
\int \sqrt{\csc (x)}(\cos (x) x-4 \sec (x) \tan (x)) \mathrm{d} x
$$

Problem 234: Unable to integrate problem.

$$
\int x^{3} \csc (x) \sec (x) \sqrt{a \sec (x)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 280 leaves, 21 steps):
$x^{3} \sqrt{a \sec (x)^{2}}+6 \mathrm{I} x^{2} \arctan \left(\mathrm{e}^{\mathrm{I} x}\right) \cos (x) \sqrt{a \sec (x)^{2}}-2 x^{3} \operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I} x}\right) \cos (x) \sqrt{a \sec (x)^{2}}+3 \mathrm{I} x^{2} \cos (x) \operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}$
$-6 \mathrm{I} x \cos (x) \operatorname{polylog}\left(2,-\mathrm{I} \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}+6 \mathrm{I} x \cos (x) \operatorname{polylog}\left(2, \mathrm{I} \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}-3 \mathrm{I} x^{2} \cos (x) \operatorname{polylog}\left(2, \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}-6 x \cos (x) \operatorname{polylog}(3$,
$\left.-\mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}+6 \cos (x) \operatorname{polylog}\left(3,-\mathrm{I} \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}-6 \cos (x) \operatorname{polylog}\left(3, \mathrm{I} \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}+6 x \cos (x) \operatorname{polylog}\left(3, \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}$
$-6 \mathrm{I} \cos (x) \operatorname{polylog}\left(4,-\mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}+6 \mathrm{I} \cos (x) \operatorname{polylog}\left(4, \mathrm{e}^{\mathrm{I} x}\right) \sqrt{a \sec (x)^{2}}$
Result(type 8, 126 leaves):

$$
2 x^{3} \sqrt{\frac{a\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}}{\left(\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}+1\right)^{2}}}+\frac{8 \mathrm{I}\left(\int \frac{x^{2} \mathrm{e}^{\mathrm{I} x}\left(3 \mathrm{I}\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}+x\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}-3 \mathrm{I}+x\right)}{4\left(\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}-1\right)\left(\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}+1\right)} \mathrm{d} x\right) \sqrt{\frac{a\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}}{\left(\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}+1\right)^{2}}}\left(\left(\mathrm{e}^{\mathrm{I} x}\right)^{2}+1\right)}{\mathrm{e}^{\mathrm{I} x}}
$$

Problem 244: Result more than twice size of optimal antiderivative.

$$
\int f^{b x+a}(\cos (d x+c)+\mathrm{I} \sin (d x+c))^{n} \mathrm{~d} x
$$

Optimal(type 3, 31 leaves, 4 steps):

$$
\frac{\left(\mathrm{e}^{\mathrm{I}(d x+c)}\right)^{n} f^{b x+a}}{\mathrm{I} d n+b \ln (f)}
$$

Result(type 3, 85 leaves):

$$
\frac{\mathrm{e}^{(b x+a) \ln (f)} \mathrm{e}\left(\frac{2 \operatorname{Itan}\left(\frac{d x}{2}+\frac{c}{2}\right)}{1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}+\frac{1-\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\right)}{\mathrm{I} d n+b \ln (f)}
$$

Test results for the 3 problems in "dog.txt"

Problem 1: Unable to integrate problem.

$$
\int F^{c(b x+a)}(f+f \sin (e x+d))^{n} \mathrm{~d} x
$$

Optimal(type 5, 107 leaves, 3 steps):

$$
-\frac{F^{c(b x+a)} \text { hypergeom }\left(\left[-2 n,-n-\frac{\mathrm{I} b c \ln (F)}{e}\right],\left[1-n-\frac{\mathrm{I} b c \ln (F)}{e}\right], \mathrm{I} \mathrm{e}^{\mathrm{I}(e x+d)}\right)(f+f \sin (e x+d))^{n}}{\left(1+\mathrm{e}^{\left.\frac{\mathrm{I}}{2(2 e x-\pi+2 d)}\right)^{2 n}}(\mathrm{I} e n-b c \ln (F))\right.}
$$

Result(type 8, 24 leaves):

$$
\int F^{c(b x+a)}(f+f \sin (e x+d))^{n} \mathrm{~d} x
$$

Problem 2: Unable to integrate problem.

$$
\int F^{c(b x+a)}(f+f \cos (e x+d))^{n} \mathrm{~d} x
$$

Optimal(type 5, 100 leaves, 3 steps):

$$
-\frac{F^{c(b x+a)}(f+f \cos (e x+d))^{n} \text { hypergeom }\left(\left[-2 n,-n-\frac{\mathrm{I} b c \ln (F)}{e}\right],\left[1-n-\frac{\mathrm{I} b c \ln (F)}{e}\right],-\mathrm{e}^{\mathrm{I}(e x+d)}\right)}{\left(\mathrm{e}^{\mathrm{I}(e x+d)}+1\right)^{2 n}(\mathrm{I} e n-b c \ln (F))}
$$

Result(type 8, 24 leaves):

$$
\int F^{c(b x+a)}(f+f \cos (e x+d))^{n} \mathrm{~d} x
$$

Problem 3: Unable to integrate problem.

$$
\int F^{c(b x+a)}(f+f \cosh (e x+d))^{n} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 3 steps):

$$
-\frac{F^{c(b x+a)}(f+f \cosh (e x+d))^{n} \text { hypergeom }\left(\left[-2 n,-n+\frac{b c \ln (F)}{e}\right],\left[1-n+\frac{b c \ln (F)}{e}\right],-\mathrm{e}^{e x+d}\right)}{\left(1+\mathrm{e}^{e x+d}\right)^{2 n}(e n-b c \ln (F))}
$$

Result(type 8, 24 leaves):

$$
\int F^{c(b x+a)}(f+f \cosh (e x+d))^{n} \mathrm{~d} x
$$

## Summary of Integration Test Results

634 integration problems


A - 379 optimal antiderivatives
B - 131 more than twice size of optimal antiderivatives
C - 10 unnecessarily complex antiderivatives
D - 108 unable to integrate problems
E - 6 integration timeouts


[^0]:    Problem 36: Attempted integration timed out after 120 seconds.

[^1]:    Result(type 3, 180 leaves):

[^2]:    Problem 53: Result more than twice size of optimal antiderivative.

[^3]:    Problem 6: Result more than twice size of optimal antiderivative.

[^4]:    Problem 24: Result more than twice size of optimal antiderivative.

[^5]:    Problem 168: Result more than twice size of optimal antiderivative.

